A Perspective on Nullators, Norators and Nullors

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In the early days of radio and line communications, the need arose for frequency-selective filters, to separate simultaneous transmissions at different frequencies.

To design these required good mathematical ability and involved complex accurate calculations. It was therefore left to 'experts' who provided the designs for practising engineers to make.

The 'theory' developed around idealised components (inductors, capacitors, resistors and ideal transformers - the last being a way to 'idealise' mutually-coupled inductance). In this idealised world, normally all was linear, time-invariant, passive, lumped, finite, and, to be useful in filter design, also usually lossless (e.g. no resistors).

This ideal world developed to become the expected foundation in the curriculum of electrical engineers and was called 'Circuit Theory' (or, by some, 'Network Theory').

The possibility that there might be other 'ideal' components arose, and a Dutchman, Tellegen, invented the gyrator, which was lossless [1].

From a gyrator and one capacitor could be made an inductor, which therefore seemed to make inductors unnecessary. (This attracted interest because practical inductors at the time were typically large, heavy and not very close to the 'ideal'). The problem was that no one knew how to make a gyrator. (The converse property, that with a gyrator and an inductor, one could make a capacitor, attracted little interest, because that did not seem to be a useful thing to do.)

Importantly all these 'ideal' components were passive. A nice result was that at two terminals brought out from a network made of a finite number of these passive components the voltage to current ratio and the current to voltage ratio (called the driving-point immittance, a generalisation of impedance and admittance) had the property of being a positive-real rational function of the complex frequency variable [2], and also, given any positive-real rational function, Bott and Duffin [3] devised a synthesis procedure which guaranteed the design of an exact implementation of this function using just the ideal R, L, C components (especially, no transformer needed, which had previously seemed often essential – as in the procedure originating with Brune). Bott and Duffin made use of an apparently unrelated mathematical theorem (the Richards Transformation [4]). In terminology that was not used until the software engineering era much later, this kind of synthesis can be described as design by an algorithm that is proved to always terminate in a finite number of steps. The availability of a synthesis procedure seemed like the 'holy grail' of engineering: to start with a specification of what was wanted, checking that it was realisable and if so to have a process which guaranteed a design which met the specification. Naturally this appealed to many educators.

With the development of thermionic vacuum-tubes (valves), amplification became possible and the idea of extending the nice ideal world of passive networks to active networks arose (with some opposition from many of the passive filter theorists). Many ideas arose for 'incorporating' activity, leading to many publications, lots of PhD's awarded, but few useful outcomes. Negative resistors

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and even negative capacitors and negative inductors were played around with and other devices invented, such as Negative Impedance Converters [5].

The Negative Impedance Converter played just a transitional role in Active RC filter design, since much better ways evolved. However, it could be considered to deserve a permanent place in Circuit Theory because of its role in completing an Abelian Group when cascading pairs of other basic two port elements. (see Table I, IT = Ideal Transformer, NIC = Negative Impedance Converter, Gyr = Gyrator, NII = Negative Impedance Inverter).

	IT	NIC	Gyr	NII
IT	IT	NIC	Gyr	NII
NIC	NIC	IT	NII	Gyr
Gyr	Gyr	NII	IT	NIC
NII	NII	Gyr	NIC	IT

Table I

Early on, Keen proposed an element which he called a 'unitor' [6, 7] to represent an idealisation of the amplifying behaviour of the triode. Tellegen also independently proposed an idealised element which was somewhat similar, though he did not give it a name [8].

Keen's idea attracted the attention of Carlin and Youla at the Polytechnic Institute of Brooklyn, who were trying to formulate a sound theoretical mathematical foundation for circuit theory [9] (to appeal to the mathematically competent but incomprehensible to most practising engineers).

From this basis, Carlin [10] proposed the nullator and the norator, two terminal elements which seemed meaningless and even stupid to most people, who described them in various curious and often inaccurate ways, mainly to give advice that they were both silly and useless. For example the nullator was said to be 'simultaneously a short and open circuit', and the norator was defined as 'nothing' because its current and voltage could have any values. In reality these descriptions were themselves inaccurate, as can be seen from Table II.

	Short-circuit	Open-circuit	nullator	norator
voltage	zero	arbitrary	zero	arbitrary
current	arbitrary	zero	zero	arbitrary

Table II

–0–	–OO–
nullator	norator
Circuit symbols for these elements	

Table III

From Table II, it is easy to arrive at some sensible statements about how these components would behave in the unlikely situation of a supply of them becoming available.

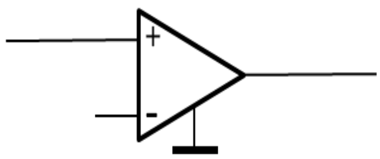
The nullator provides an 'impossible' constraint when connected in an otherwise 'normal' circuit, and the norator does nothing at all.

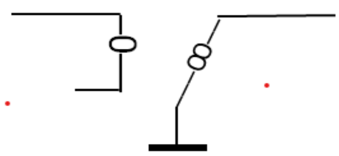
Slight discomfort with these observations could arise because of the claim that with a gyrator and positive and negative resistors, it is possible to 'make' a nullator, and circuit diagrams were published for this. It is questionable whether these are valid in any rigorous way, and may be considered analogous to those mathematical 'proofs' which start with some known agreed axioms, and go step by step using easy standard mathematics and reach a result such as '1=2'. Everyone knows that these contain, at some step, a hidden, non-obvious, but usually simple mistake, designed to fool the reader. (by contrast a proof that '1 + 2 = 0' may be valid if working within a finite field)

While the nullator and norator individually can be considered useless, the 'impossible constraint' imposed by the connection of the nullator can be 'relieved' or 'resolved' by the connection of a norator elsewhere, the pair becoming known as the nullor. Particularly in the form of a three-terminal component (e.g. one terminal shared by both nullator and norator) the nullor turns out to be essentially the same as the 'unitor' and can be used as an idealised model for a triode used as a grounded cathode amplifier or a transistor used as a grounded emitter amplifier (or even an FET used as a common source amplifier). To be more precise, the nullor corresponds to a triode with small-signal parameters g_a = zero and $g_m = \infty$, and to a bipolar transistor for which $h_{ie} = 0$ and $h_{fe} = \infty$. In the design of analogue bipolar transistor feedback amplifiers, it was typical to assume that the base current and base emitter voltages were zero, so making emitter and collector currents equal. This corresponds exactly with representing the transistor as a nullor. This design assumption has the advantage of making designs which are robust against transistor parameter tolerances.

Much later, when the Fairchild 741 integrated circuit OpAmp became available as a cheap widely used electronic component [11], the nullor was realised to be, in effect, an excellent representation of an 'ideal OpAmp' and provided a mechanism to simplify the nodal analysis of R,L,C circuits with OpAmps [12, 13].

When the nullor is used to represent a balanced differential-input OpAmp, the norator part does not share a terminal with the nullor part, and one of its terminals connects directly to ground. This is an improvement over the common practice in circuit diagrams incorporating differential-input OpAmps, where this ground connection is often omitted, creating a component which then apparently does not comply with Kirchhoff's Current Law (e.g. a triangular symbol where no current flows at the + and – input terminals but current does flow at the output terminal)





OpAmp circuit symbol	Nullor model of ideal OpAmp
Nullor equivalence to differential-input OpAmp	

So, initially, the nullor created a good way to represent an ideal OpAmp. Later the opposite approach arose: how to make an integrated circuit which was a good implementation of a nullor [14, 15, 16].

Further versatility is provided by the ability to replace all the controlled sources by combinations of nullors and resistors, which can provide simplicity in setting up the equations for network analysis [17].

A very thorough treatment of the ways that nullors can be used in RC-Active Circuits is provided by Bruton [18].

Circuit Theory also incorporated study of the means of analysing interconnected ideal components: That is, given a network of N nodes and B branches, writing a set of linear simultaneous equations from which all voltages and all currents could be calculated, and doing this in various ways. Nodal analysis required N-1 equations, and was usually best for electronic circuitry. Presence of nullors enabled the number of equations to be reduced by one per nullor, which was a significant advantage with the computational aids available at that time.

Inclusion of some of this material in the curriculum for electrical and electronics engineers seems valuable, although the overfull present-day curriculum is such that it is implicit in this suggestion that some other material needs to be removed.

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