A Note on Simultaneous Choice-free Synthesis

— For Maciej Koutny on the occasion of his 60'th birthday —

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Abstract. A finite set of transition systems $\{TS_1, \ldots, TS_n\}$ shall be called *simultaneously Petri net solvable* if there is a single Petri net N with different initial markings $\{M_{01}, \ldots, M_{0n}\}$, such that for every $i = 1, \ldots, n$, the reachability graph of (N, M_{0i}) is isomorphic to TS_i. We consider the special case that N must be *choice-free*, that is, contains no structural choices, and we explore how the analysis of small cycles allows to discard ill-formed systems in a pre-synthesis phase, and how they can be used to construct an adequate solution when this is possible.

Keywords: Choice-Freeness, Initial Markings, Labelled Transition Systems, Petri Nets, System Synthesis.

1 Introduction

In various papers, Maciej Koutny developed – with some colleagues – some kind of event structures allowing to model, in particular, a *simultaneity* feature. Thus, Maciej seems to like this word. This prompted the second author of this paper to consider introducing a new kind of simultaneity in our area of research. The idea is to allow, in a Petri net synthesis context, an underlying net to have a set of initial markings, rather than only a single one.

Classically, a Petri net synthesis problem starts from a finite labelled transition system (lts) and asks to build an unlabelled Petri net (possibly of some structural subclass) and an initial marking such that the corresponding reachability graph is isomorphic to the given lts. If this is not possible, one would like to have reasons why, as simple as possible, to be able to mend the given lts in order to finally reach a solvable case satisfying a more or less specific aim.

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In a simultaneous synthesis, one starts from several lts and one searches for a single net and several initial markings such the corresponding reachability graphs are isomorphic to the given lts's.

As a motivating example, consider the labelled transition systems shown in Figure 1. Note that their label sets are not equal. We shall be asking the question whether or not there exists a single Petri net N with two initial markings M_{01}, M_{02}, such that the two transition systems are implemented by (N, M_{01}) and by (N, M_{02}), respectively. Figure 2 shows that this is indeed possible. In this case, we say that N, together with the two initial markings, *solves* – or *synthesises* – the two given transition systems *simultaneously*.

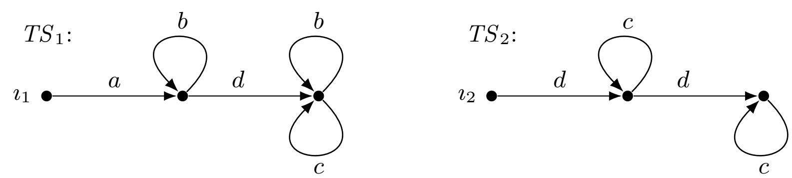

Fig. 1. Two finite transition systems TS_1 and TS_2. A simultaneous Petri net solution is sought.

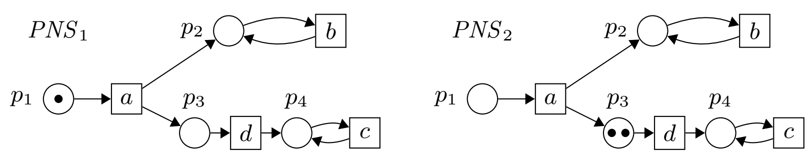

Fig. 2. PNS_1 and PNS_2 simultaneously solve TS_1 and TS_2 (respectively) choice-freely (no place has more than one output transition), since the reachability graph of PNS_i is isomorphic to TS_i $(i = 1, 2)$. Observe that PNS_1 and PNS_2 have the same set of transitions, although not all of them occur in both reachability graphs.

The choice of a specific subclass of nets to be built may considerably change the problem, and its complexity, as illustrated in Figure 3. This example shows that even if TS_3 and TS_4 are individually solvable choice-freely and a simultaneous solution exists, there does not have to be a simultaneous choice-free solution.

Definition 1. (SIMULTANEOUS) SOLVABILITY OF *lts* BY *PNS*

An (unlabelled) Petri net N and a marking M_0 solve (or synthesise) a labelled transition system TS if the reachability graph of the Petri net system (N, M_0) is isomorphic to TS. A Petri net N and n markings $M_{01}, \ldots, M_{0n}$ solve n labelled

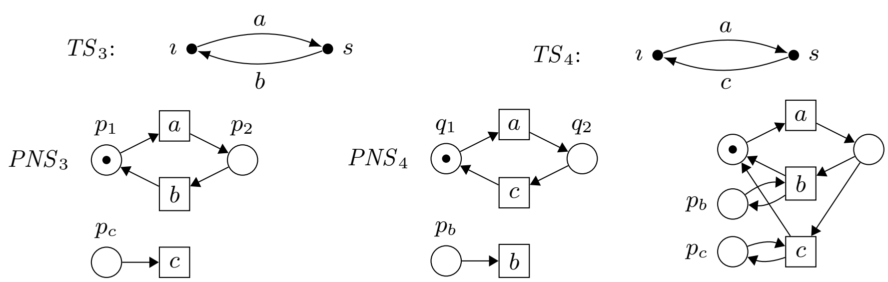

Fig. 3. Two transition systems TS_3 and TS_4, and individual choice-free solutions of them (PNS_3, respectively, PNS_4), both over the transition set $\{a, b, c\}$. There exists a simultaneous solution (bottom right): depending on whether p_b or p_c carries an initial token, TS_3 or TS_4 is solved. Later, we will demonstrate that no choice-free simultaneous solution exists.

transition systems $TS_1, \ldots, TS_n$ simultaneously if the reachability graph of the Petri net system (N, M_{0i}) is isomorphic to TS_i, for $1 \leq i \leq n$. A transition system is solvable (choice-freely solvable, or cf-solvable, for short) if it can be solved by a Petri net system (a choice-free Petri net system, respectively), and similarly for simultaneous solvability. □ 1

In order to solve such a problem, we could use an additional initial state from which n arrows (with fresh labels) lead to the n initial states of $TS_1, \ldots, TS_n$, and ask for the individual solvability of the resulting amended system. Then one considers the markings reached after performing the additional transitions, and drops the latter, in order to obtain a simultaneous solution. However, this general procedure is not very effective because the complexity of the synthesis algorithms usually grows rapidly with the number of states in the lts. Moreover it does not work for searching a choice-free solution if $n \geq 2$.

We shall thus adopt a more “distributed” strategy, considering the various lts separately before consolidating their characteristics.

2 Small cycles, semiflows and residues

Previous works on cf-synthesis exhibited the importance of Parikh vectors of sequences in an lts (counting the number of occurrences of each label), small cycles (no other cycle has a smaller Parikh vector) and short paths between two states (no shorter path exists).

A well-known property of bounded choice-free systems is that any two small cycles in its reachability graph either have the same Parikh vector or disjoint supports. This is illustrated by PNS_1 and TS_1 in Figures 2 and 1, where there are two disjoint Parikh vectors of small cycles (b and c).

This is no longer true for unbounded systems, as illustrated by Figure 4.

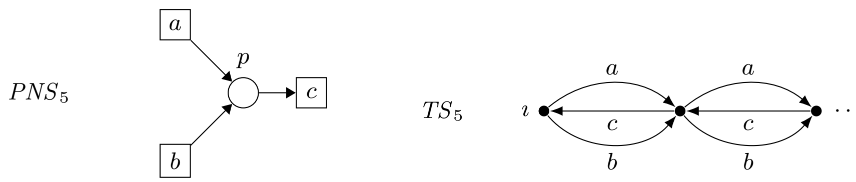

Fig. 4. An unbounded cf-net (on the left) and its reachability graph (on the right) with intersecting small cycles ac and bc.

A first result we obtained is that the disjunction property extends to small cycles in several bounded reachability graphs of a same cf-net.

Theorem 1. Small cycles in a simultaneous synthesis
Let (N, M_0^1) and (N, M_0^2) be two bounded systems arising from the same choice-free net N. Let $M_1[\sigma_1\rangle M_1$ be a small cycle in the reachability graph of the former and $M_2[\sigma_2\rangle M_2$ be a small cycle in the reachability graph of the latter. Then σ_1 and σ_2 are either Parikh-equivalent or disjoint.

This allows to rule out the simultaneous solvability of systems TS_3 and TS_4 in Figure 3 since the two small cycles ab and ac are neither Parikh-equivalent nor disjoint.

In fact, it can also be proved that the Parikh vectors of small cycles occurring in the finite reachability graph of some cf-net are minimal semi-flows of the latter, and the next series of new results concern semiflows. In formulating them, we exploit a classic residue operation defined for sequences and extended to T-vectors.

Let $\tau, \sigma \in T^*$ be two sequences over some label set T. The (left) residue of τ with respect to σ, denoted by $\tau \stackrel{\bullet}{-} \sigma$, arises from cancelling successively in τ the leftmost occurrences of all symbols from σ, read from left to right. Inductively: $\tau \stackrel{\bullet}{-} \varepsilon = \tau$; $\tau \stackrel{\bullet}{-} t = \tau$ if $t \notin supp(\tau)$; $\tau \stackrel{\bullet}{-} t$ is the sequence obtained by erasing the leftmost t in τ if $t \in supp(\tau)$; and $\tau \stackrel{\bullet}{-} (t\sigma) = (\tau \stackrel{\bullet}{-} t) \stackrel{\bullet}{-} \sigma$. Said differently, $\tau \stackrel{\bullet}{-} \sigma$ is τ with, for each $t \in T$, the first $\min(\mathcal{P}(\tau)(t), \mathcal{P}(\sigma)(t))$ occurrences of t dropped. Let $\sigma \in T^*$ and $\Phi \in \mathbb{N}^T$: $\sigma \stackrel{\bullet}{-} \Phi$ is the sequence obtained from σ by cancelling the $\min(\mathcal{P}(\tau)(t), \Phi(t))$ leftmost occurrences of t for each $t \in T$. Let $\Phi, \Psi \in \mathbb{N}^T$: $\Psi \stackrel{\bullet}{-} \Phi$ is the T-vector such that, for each $t \in T$, $(\Psi \stackrel{\bullet}{-} \Phi)(t) = \max(\Psi(t) - \Phi(t), 0) = \Psi(t) - \min(\Psi(t), \Phi(t))$. The consistency between these various forms of residues arises from the observation that $\mathcal{P}(\tau \stackrel{\bullet}{-} \sigma) = \mathcal{P}(\tau \stackrel{\bullet}{-} \mathcal{P}(\sigma)) = \mathcal{P}(\tau) \stackrel{\bullet}{-} \mathcal{P}(\sigma)$. Other interesting properties about residues are that $(\sigma \stackrel{\bullet}{-} \sigma_1) \stackrel{\bullet}{-} \sigma_2 = \sigma \stackrel{\bullet}{-} (\mathcal{P}(\sigma_1) + \mathcal{P}(\sigma_2)) = (\sigma \stackrel{\bullet}{-} \sigma_2) \stackrel{\bullet}{-} \sigma_1$ and $\sigma\sigma' \stackrel{\bullet}{-} \sigma = \sigma'$.

In the reachability graph of a cf-system (N, M_0), it is known that all the short paths from M_0 to a reachable marking M have the same Parikh vector, called the distance Δ_M of M from the initial marking.

Theorem 2. Properties of semiflows in cf-systems

Let (N, M_0) be a cf-system, M any reachable marking and Ψ a semiflow of N. Then,

1. *$\Delta_M \not\geq \Psi$ (short distance property);*
2. *for some reachable marking M', we have $\Delta_{M'} = \Delta_M \stackrel{\bullet}{-} \Psi$ (reduced distance property);*
3. *if there is a cycle around M with Parikh vector Γ disjoint from Ψ, there is also a cycle around M' with Parikh vector Γ when $\Delta_{M'} = \Delta_M \stackrel{\bullet}{-} \Psi$ (early cycle property).*

This may be the base for the development of very discriminating structural checks during an (individual or simultaneous) cf-presynthesis. For instance, Figure 5 illustrates the use of the short distance property, Figure 6 illustrates the reduced distance property, and Figure 7 illustrates the early cycle property

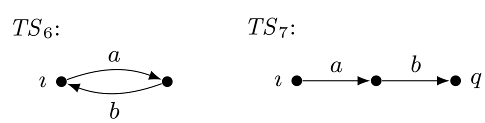

Fig. 5. Two transition systems TS_6 and TS_7 which are individually, but not simultaneously, solvable by a choice-free net. In TS_6, there is a small cycle $\imath[ab\rangle\imath$, while in TS_7, $\Delta_q = \mathcal{P}(ab)$ for a state $q \neq \imath$, so that $\neg(\Delta_q \not\geq \mathcal{P}(ab))$, contradicting the short distance property.

3 Simultaneous cf-synthesis

If a set of transition systems has a simultaneous cf-solution, then necessarily the pre-synthesis, which checks the properties described above, succeeds, and all given transition systems have an individual solution. Conversely:

1. If the pre-synthesis fails, we know there is no solution and we should not enter the proper synthesis phase.
2. If the pre-synthesis succeeds, it may still happen that some (or all) given transition systems have no individual solution.
3. If the pre-synthesis succeeds and all given transition systems have an individual solution, it may still happen that there is no simultaneous solution.

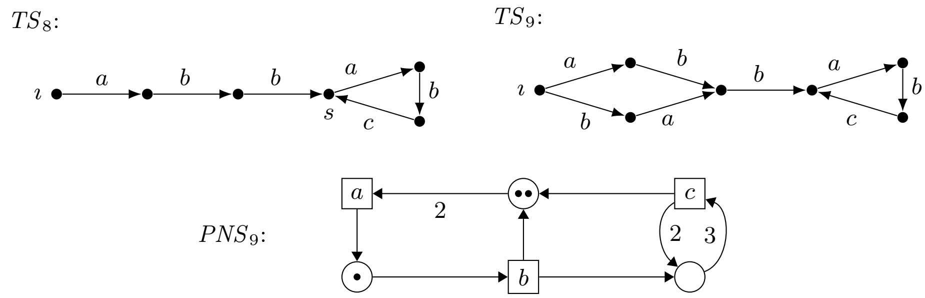

Fig. 6. TS_8 does not satisfy the reduced distance property since $\Delta_s = \mathcal{P}(abb)$, there is a small cycle $s[abc\rangle s$, but there is no state s' such that $\Delta_{s'} = \mathcal{P}(abb \mathbin{\dot{-}} abc) = \mathcal{P}(b)$. Hence, it is not (individually) cf-solvable. By contrast, TS_9 satisfies the reduced distance property and is cf-solvable, as illustrated by PNS_9.

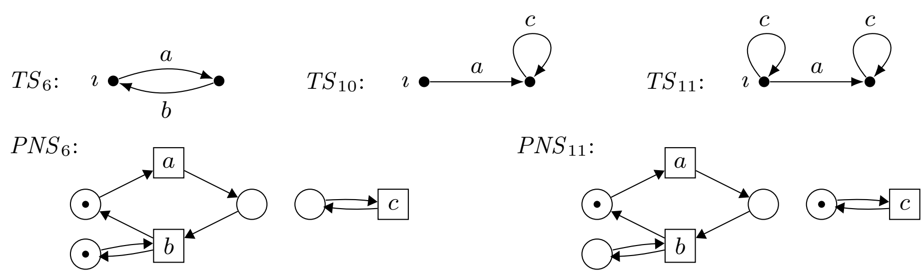

Fig. 7. The two transition systems TS_{10} and TS_{11} are individually cf-solvable. TS_{10} is not simultaneously cf-solvable with TS_6, since there is a loop c after a, but not initially while $a \mathbin{\dot{-}} ab = \varepsilon$, contravening the early cycle property. By contrast, TS_{11} is simultaneously cf-solvable with TS_6, as shown by PNS_6 and PNS_{11}.

4. If there is one, it is not always trivial to build it from the obtained individual ones. This is why we need to proceed adequately, as explained by means of Figures 8 and 9.

If there is a simultaneous solution, the underlying net must have as (minimal) semiflows the Parikh vectors of all the (small) cycles in all the given lts (possibly more). The classic pre-synthesis procedures developed in previous papers provide for each given lts TS_i the set $\mathcal{G}_i$ of all the Parikh vectors of small cycles. We may then define $\mathcal{G} = \cup_i \mathcal{G}_i$. The synthesis procedures presented in the same papers build for each TS_i (when it is possible) a cf-solution PNS_i which is automatically compatible with all the semiflows in $\mathcal{G}_i$. It is however possible to adapt those synthesis procedures in order to build (when possible) for each TS_i a cf-solution PNS'_i compatible with all the semiflows in $\mathcal{G}$. If this is not possible, we know there is no simultaneous cf-solution. The main result we obtain is then:

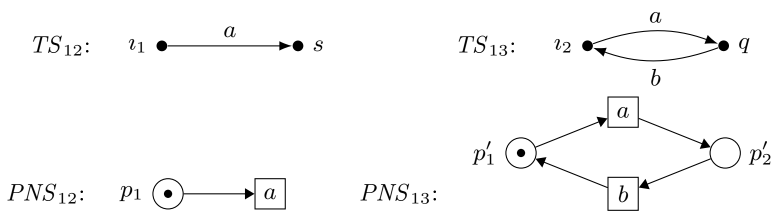

Fig. 8. Two transition systems TS_{12} and TS_{13} and two individual cf-solutions: it is not easy to deduce a simultaneous solution (if possible).

Proposition 1. Transition homogenisation and merging

Let $PNS'_1, \ldots, PNS'_n$ be solutions of $TS_1, \ldots, TS_n$ (respectively), all with the same transition set T, while respecting all the semiflows in $\mathcal{G}$; then in their synchronisation it is possible, for all $i \in \{1, \ldots, m\}$, to choose a marking in order to generate TS_i.

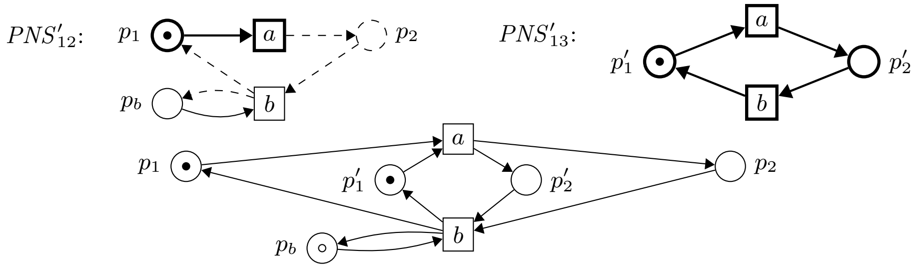

Fig. 9. Homogenised individual cf-solutions of TS_{12} and TS_{13}. Compared with PNS_{12} in Figure 8, the net PNS'_{12} also contains a transition b with a token-free input place p_b (in order to make the label sets equal), as well as dashed arrows and a new place p_2. The latter create a semiflow which corresponds to the small cycle of TS_{13} (and thus to the semiflow (a, b) of PNS_{13} and PNS'_{13}). The simultaneous solution arises by transition synchronisation (with markings according to Proposition 1) and is shown at the bottom of the figure. Depending on whether a token is absent or not on p_b, TS_{12} or TS_{13} is solved.

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