



Async-Analog: Happy Cross-talking?



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async.org.uk; workcraft.org

Vienna is now async Capital

- **A bit of history:**
 - **DDECS'10: “Async: Quo vadis?”, Eyjafjallajökull volcano, getting stuck in Vienna**
 - **Async design course in 2011, Belvedere**
 - **Async 2013 – 4 papers from TU Vienna accepted**

And finally, the whole of ASYNC is here!

Agenda

- **Prologue:**
 - **Why, where and how A and A meet**
- **Analog methods for Async circuits**
- **A4A: Async design for Analog electronics**
 - **Design flow for A4A**
 - **Workcraft.org**
- **AxA: Mixed-signal design with Async control**
 - **Analog-Async Co-design flow**
 - **Workcraft and LEMA**
- **Epilogue:**
 - **Messages and Open challenges**

Key message from our experience

- **PRODUCTIVITY is a King**

Although

- **Robustness, Low Power, Performance ... maybe a Queen**

First you show the Queen and then you push the King!



Prologue

Prologue

**Async people: How and where do you face with
analog?**

**Analog people: How and where do you face with
digital (or even async)?**

Analog behaviour

- Typically in continuous time and level
- Can be represented by EM forces/fields, electric charge, magnetic flux, voltage, current
- In various applications can represent mechanical, chemical, thermal etc. forms of information or energy
- Dynamics can be represented in time and frequency domains
- Typically separates signal flows between “data” and power supply but does not always need to
- Applications: sensing, measurement, signal processing, control, power
- Interfaces between digital and analog: analog elements inside digital components, ADC and DACs, digital control of analog

Analog elements

- **Amplifiers:**
 - **Operational amplifiers**
 - **Low noise amplifiers**
 - **Sense amplifiers**
- **Current mirrors**
- **Bandgap circuits**
- **Delay elements**
- **Oscillators**
- **Transmission lines**

Async (digital) behaviour

- Triggered by events (e.g. level-crossing)
- Modelled by
 - cause-effect relations
 - token flow
 - handshakes
 - data-flow
- Power-driven timing
- Applications: interfacing, control, pipeline

Question to think about:

**Do we need to divide the world into
analog and digital?**

*Async helps us to remove or at least lower the A-to-D
and D-to-A walls*

Question to think about:

**Do we need to divide the world into
analog and digital?**

Async helps us to remove or at least lower the A-
to-D and D-to-A walls

Let's think about other dichotomies, maybe
Electronic to Bio?

But better replace “Divide and Conquer” with
“Unite and Survive”!

Analog for Asynchronous

Didn't we do well with this!?

Analog for Async

Analog circuit design and analysis has been helping asynchronous design for ages:

- Time comparison (arbiters and synchronizers)**
- Metastability analysis**
- Noise and Cross-talk analysis**
- EMI analysis and optimisation**
- Timing optimisation, e.g. Surfing**
- ...**

Work by Molnar, Kinniment, Greenstreet, Ginosar and others

Why and what is timing comparison?

This is probably the oldest story of analog serving for async (and digital as a whole)

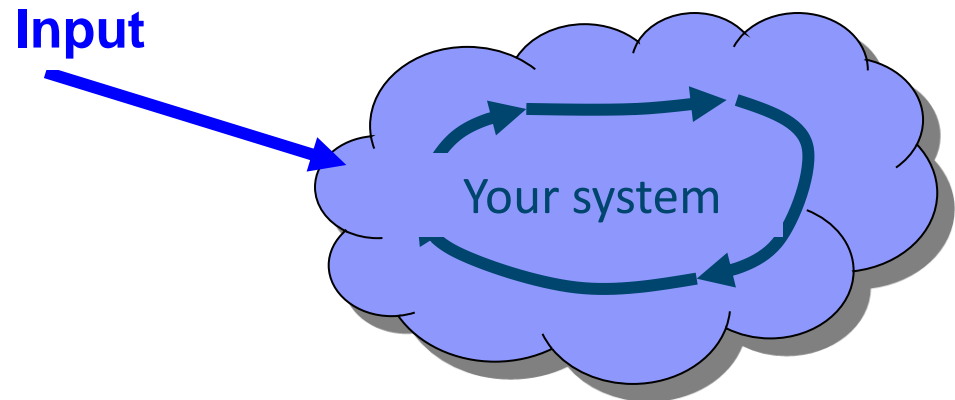


Telling if some event happened before another event

Synchronizers and arbiters

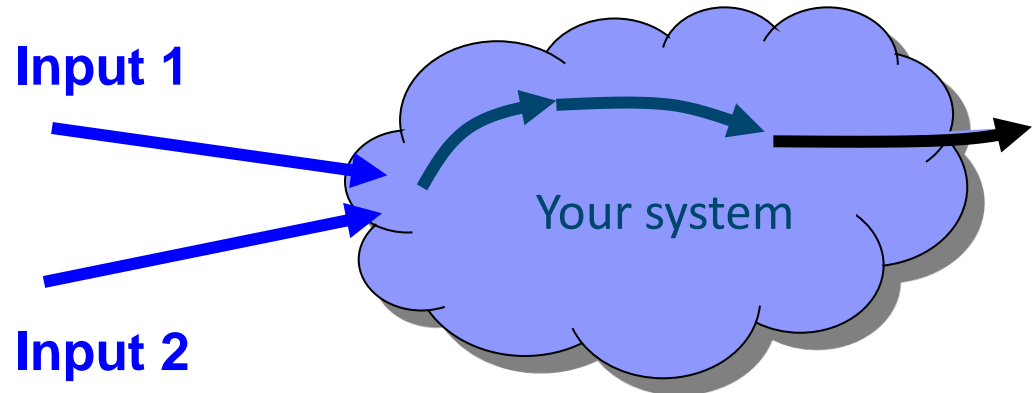
- Synchronizer

Decides which clock cycle to use for the input data



- Asynchronous arbiter

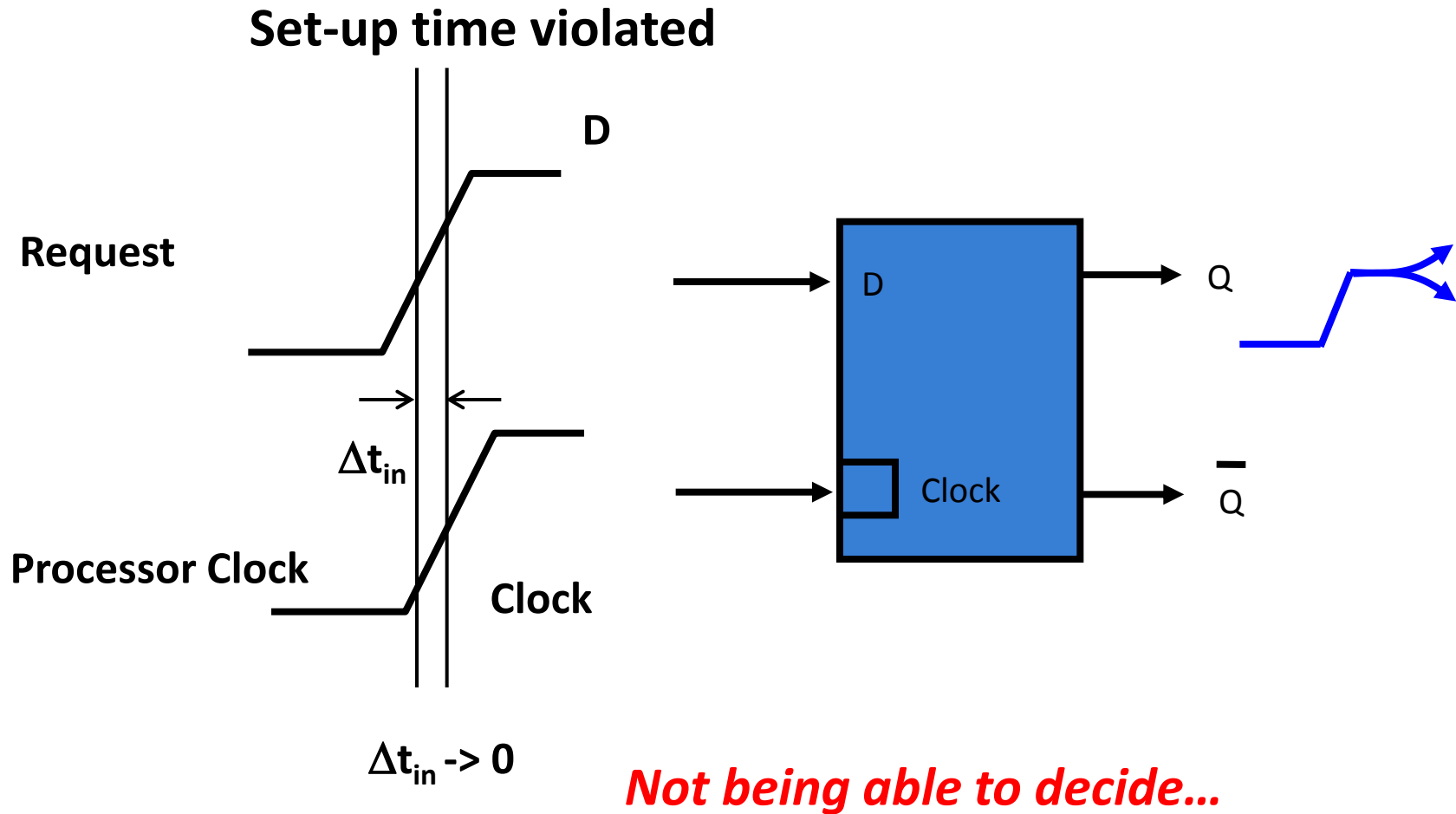
Decides the order of inputs



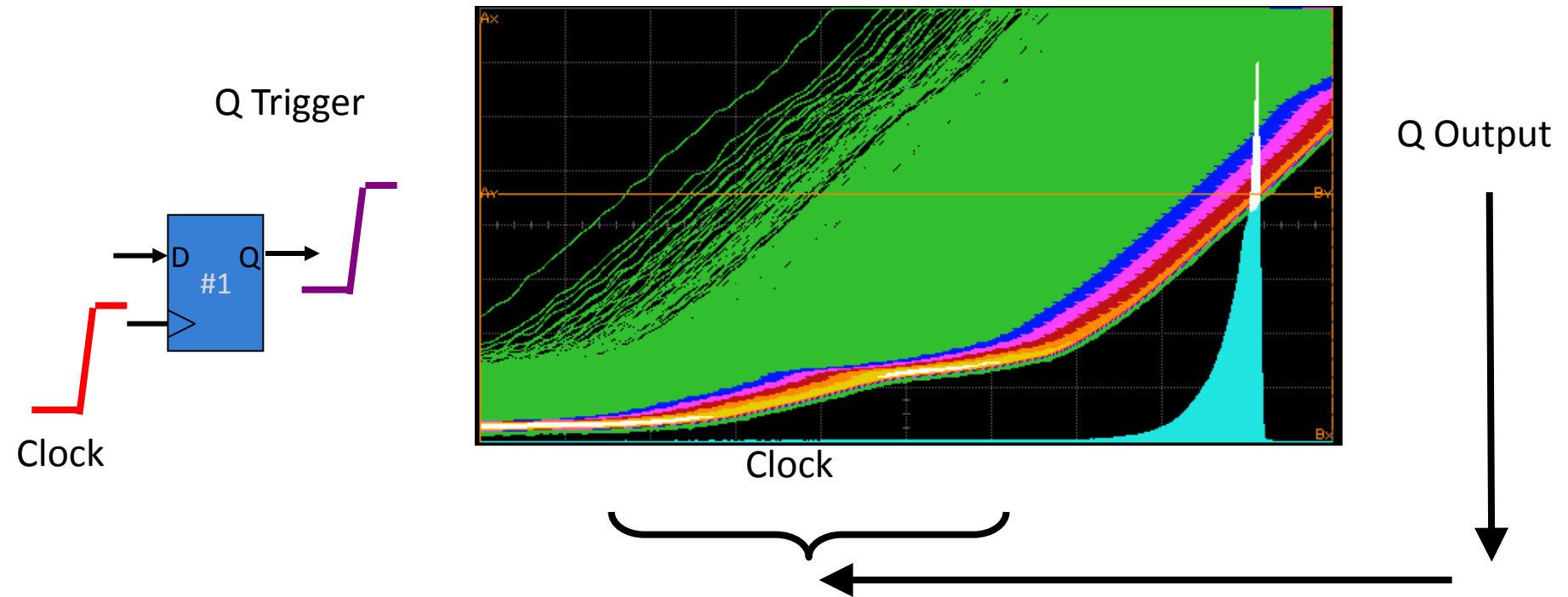
Time Comparison Hardware

- **Digital comparison hardware (which compares integers) is easy**
 - Fast
 - Bounded time
- **Analog comparison hardware (which compares reals like time) is hard**
 - Normally fast, but takes longer as the difference becomes smaller
 - Can take forever
- **Synchronization and arbitration involve comparison of time**

Metastability is....

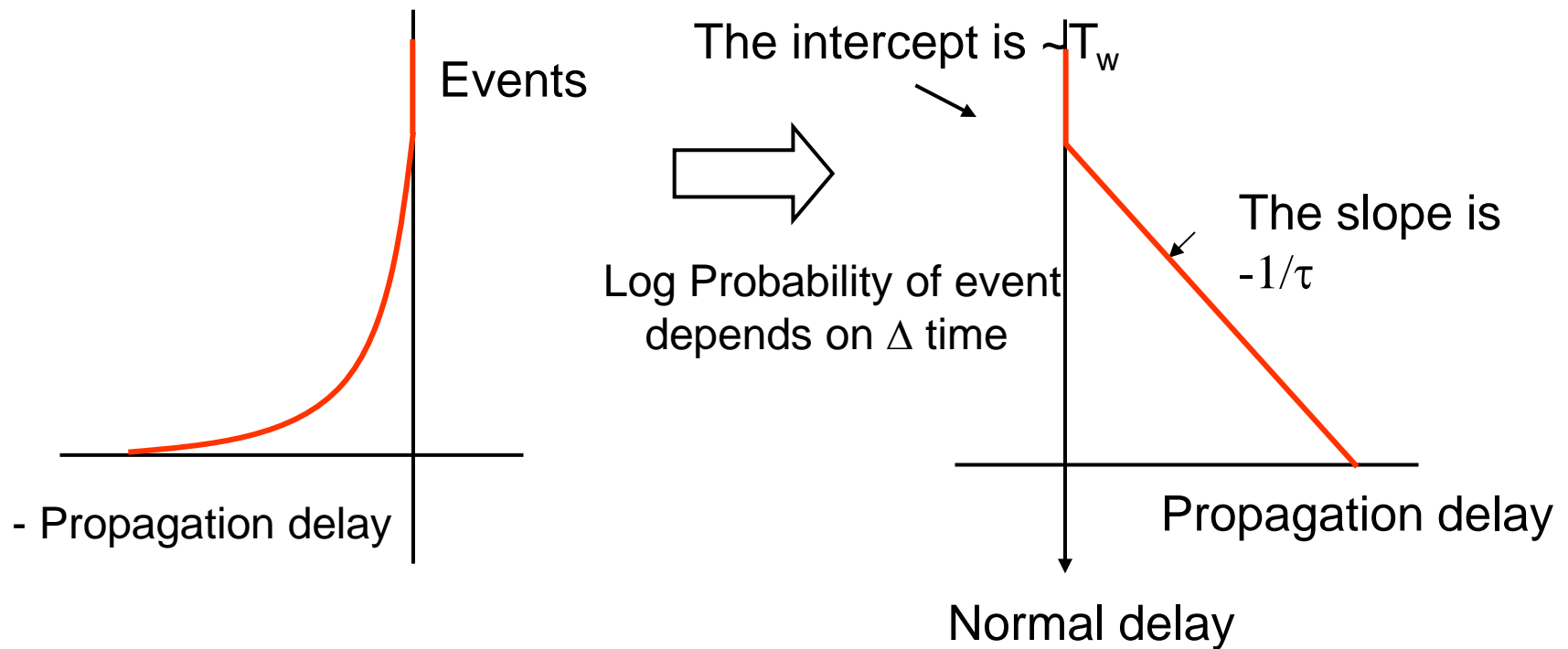


Typical responses

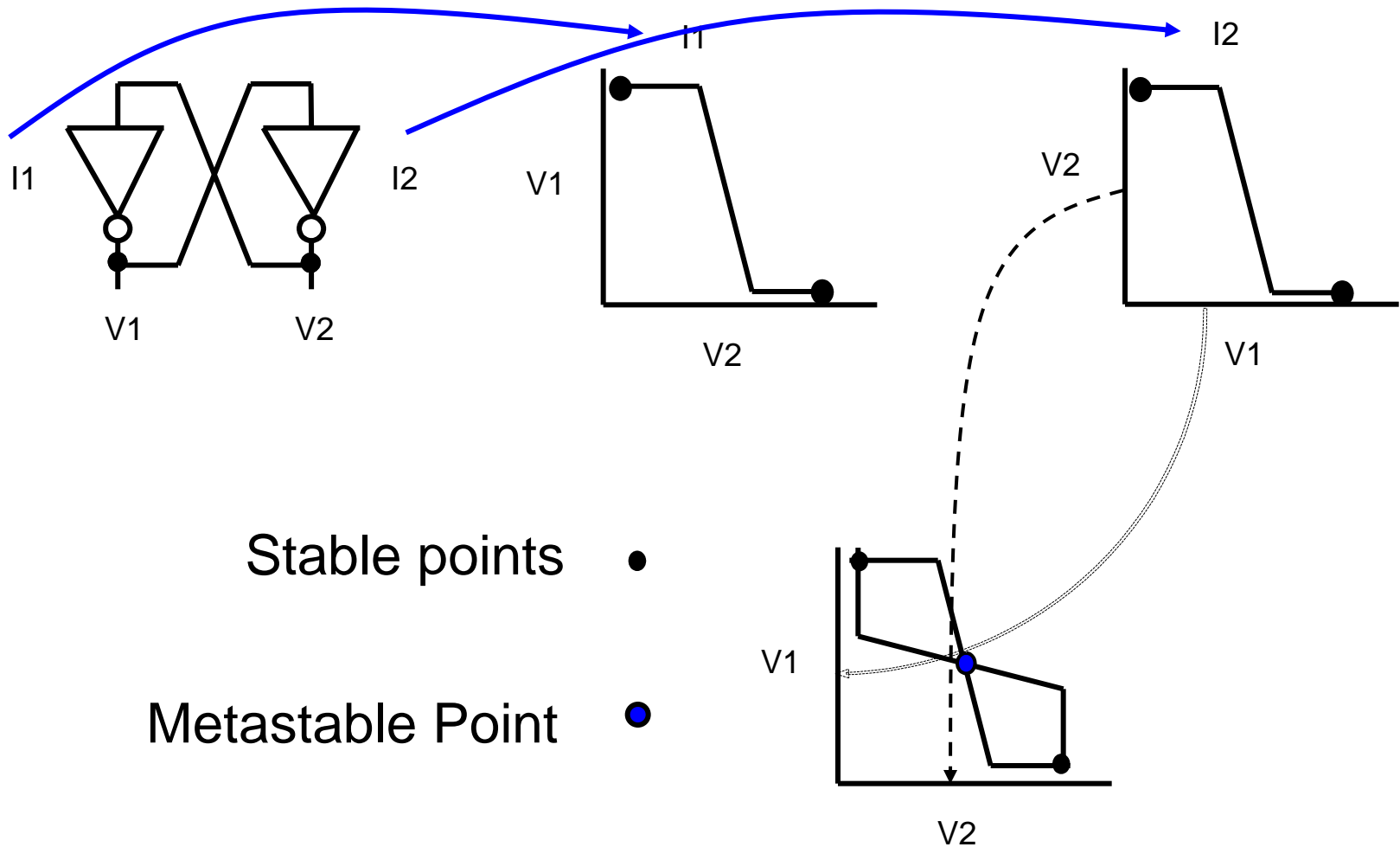


- We assume all starting points are equally probable
- Most are a long way from the “balance point”
- A few are very close and take a long time to resolve

Event Histogram

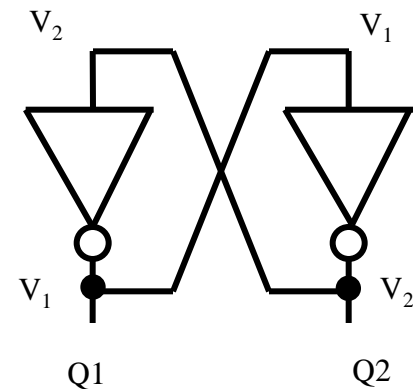
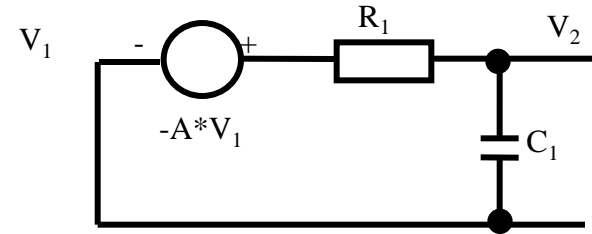


Metastability in a Latch



Linear Model

- Simple linear model leads to two exponentials
- τ_a is convergent, τ_b is divergent



$$\tau_1 = \frac{C_1 \cdot R_1}{A}, \tau_2 = \frac{C_2 \cdot R_2}{A} \quad g_m = \frac{A}{R}$$

$$0 = \tau_1 \cdot \tau_2 \cdot \frac{d^2 V_1}{dt^2} + \frac{(\tau_1 + \tau_2)}{A} \cdot \frac{dV_1}{dt} + \left(\frac{1}{A^2} - 1\right) \cdot V_1$$

$$V_1 = K_a \cdot e^{\frac{-t}{\tau_a}} + K_b \cdot e^{\frac{t}{\tau_b}}$$

How often does it fail?

- The output trajectory is an exponential that depends on the starting condition K , K depends on Δt_{in}
- Suppose the clock frequency is f_c , the data rate f_d and $K_a = 0$
- In M seconds we have $M \cdot f_c$ clocks.
- The probability of a data change within Δt_{in} of any clock is $\Delta t_{in} \cdot f_d$, so there will be one within M seconds if
- The time taken to resolve this event is t (T_w is the metastability window)

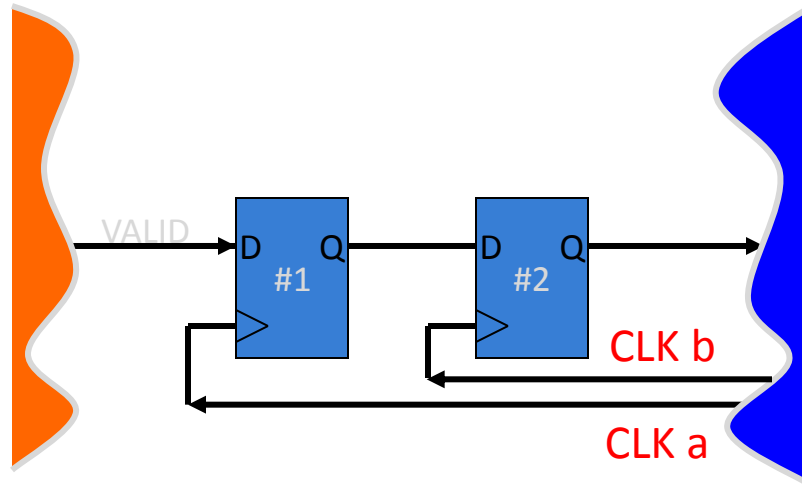
$$V = K \cdot e^{\frac{t}{\tau}}$$

$$\Delta t_{in} = \frac{1}{M \cdot f_c \cdot f_d}$$

$$\frac{V}{K} = \frac{T_w}{\Delta t_{in}} = e^{\frac{t}{\tau}}$$

Synchronizer

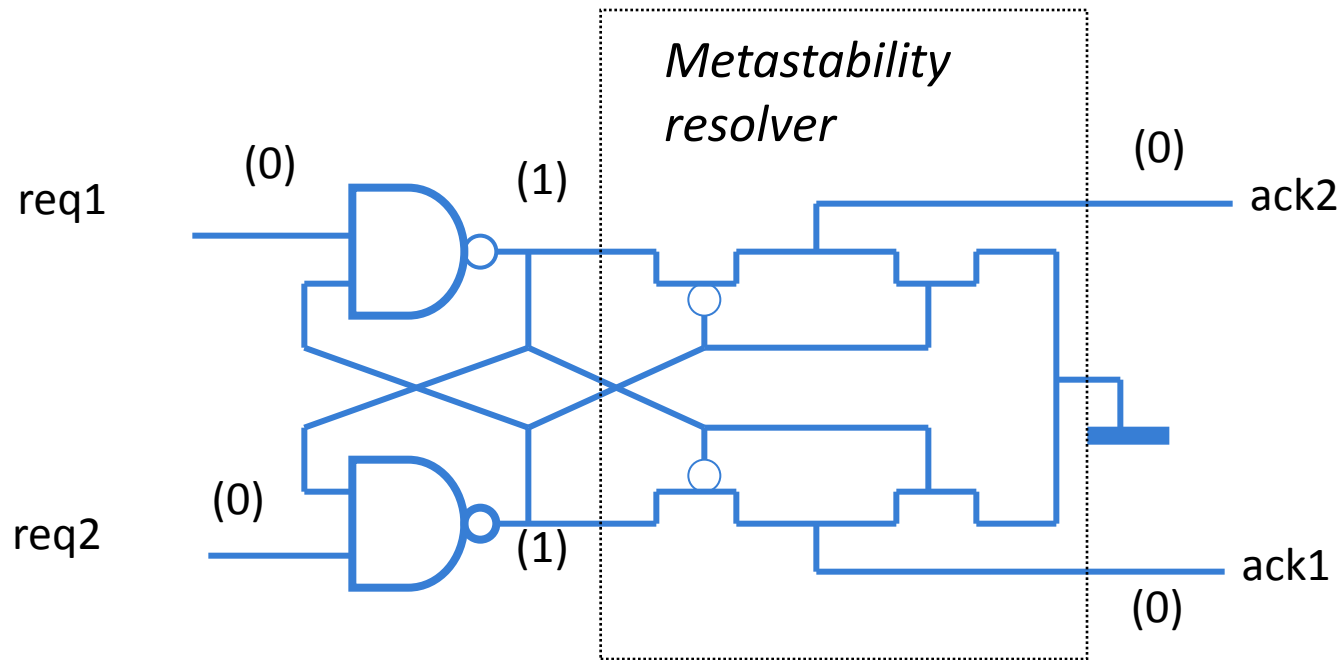
- t is time allowed for the Q to change between CLK a and CLK b
- τ is the recovery time constant, usually the gain-bandwidth of the circuit
- T_w is the “metastability window” (aperture around clock edge in which the capture of data edge causes a delay that is greater than normal propagation delay of the FF)
- τ and T_w depend on the circuit
- We assume that all values of Δt_{in} are equally probable



$$MTBF = \frac{e^{t/\tau}}{T_w \cdot f_c \cdot f_d}$$

Two-way arbiter (Mutual exclusion element)

Basic arbitration element: Mutex (due to Seitz, 1979)



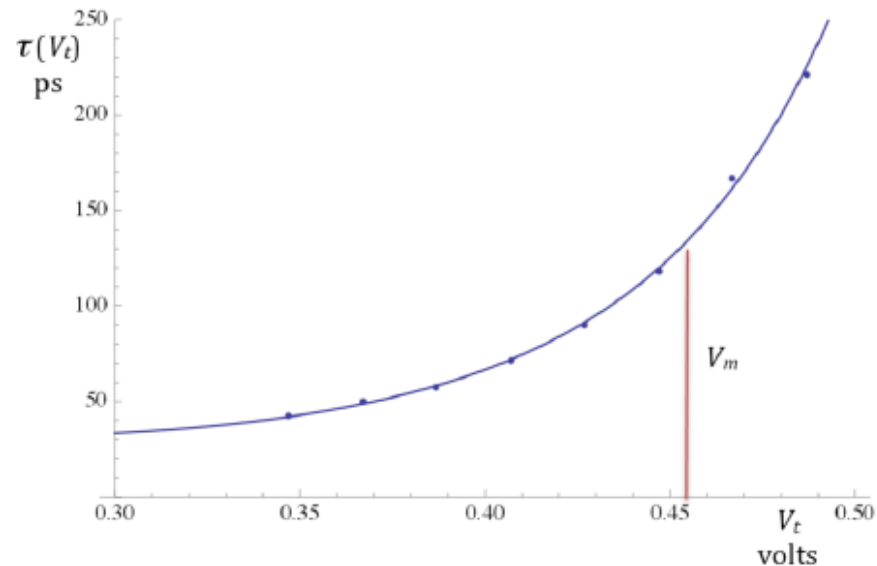
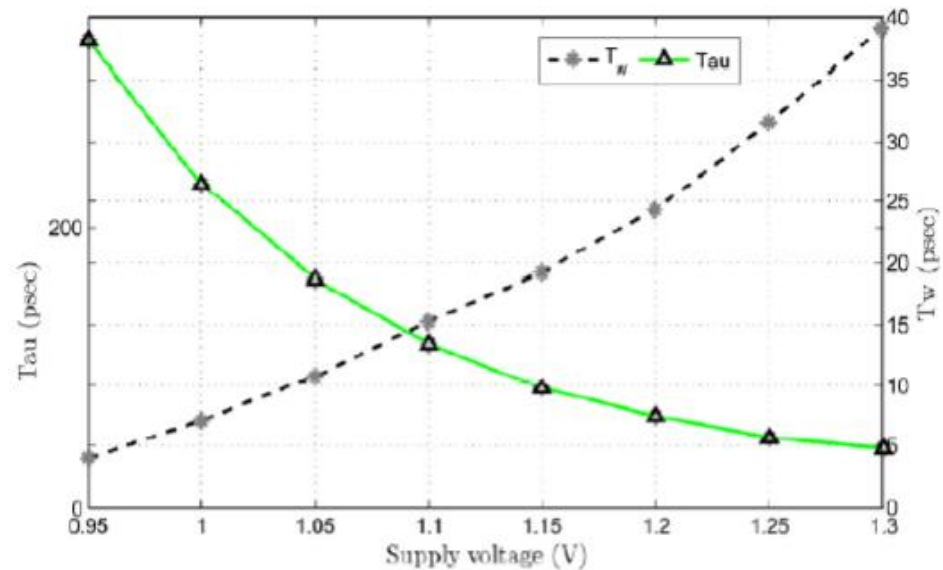
This is a key building block for multi-way arbiters, time-to-digital converters, GALS controllers, analog to async interfaces ...

Metastability analysis

- Characterisation of Δt_{in} , t , τ and MTBF, in relation to the technology parameters and V_{dd}
- This requires extensive simulation effort
- Techniques such as bisection with numerical integration can be applied:
 - Timing interval around metastability point is minimized obtaining progressively smaller voltage window sizes and their corresponding (larger) settling times (Yang-Greenstreet)
 - Other analog methods for reducing simulation time are used, such as voltage transfer curves and current compensation (Beer-Ginosar)
 - Blendics has tools which use some of these methods
 - Time-dependent gain analysis in multi-stage REAL synchronizers (Reiher-Greenstreet-Jones – this ASYNC!)

What else affects tau

- Vdd matters
- Feature size
- Process variability (V_t)
- Duty cycle of clock (for master-slave FF based)
- Multi-stage synchronizers
- Data and synchronizer FFs are totally different



Source: Blendics

Other areas for analog for async

- **Logic cell design:**
 - GasP
- **Power supply:**
 - IR degradation,
 - Drooping,
 - Subthreshold
- **Interconnect:**
 - Transmission line models,
 - Cross-talk and noise
- **Delay analysis and design:**
 - stray, inertial, pure delays

Applying analog knowledge has always helped in:

- Speeding circuits up,
- Reducing power,
- Improving reliability and robustness

**What about the other direction:
Async for Analog?**



Motivation for Async for Analog

- Analog and Mixed Signal (AMS) design becomes more complex:
 - More functionality
 - Move to deep submicron after all!
- According to Andrew Talbot from Intel (2016)
“transistors are very fast switches, netlists are huge, parasitics are phenomenally difficult to estimate, passives don’t follow Moore’s law, reliability is a brand new landscape”

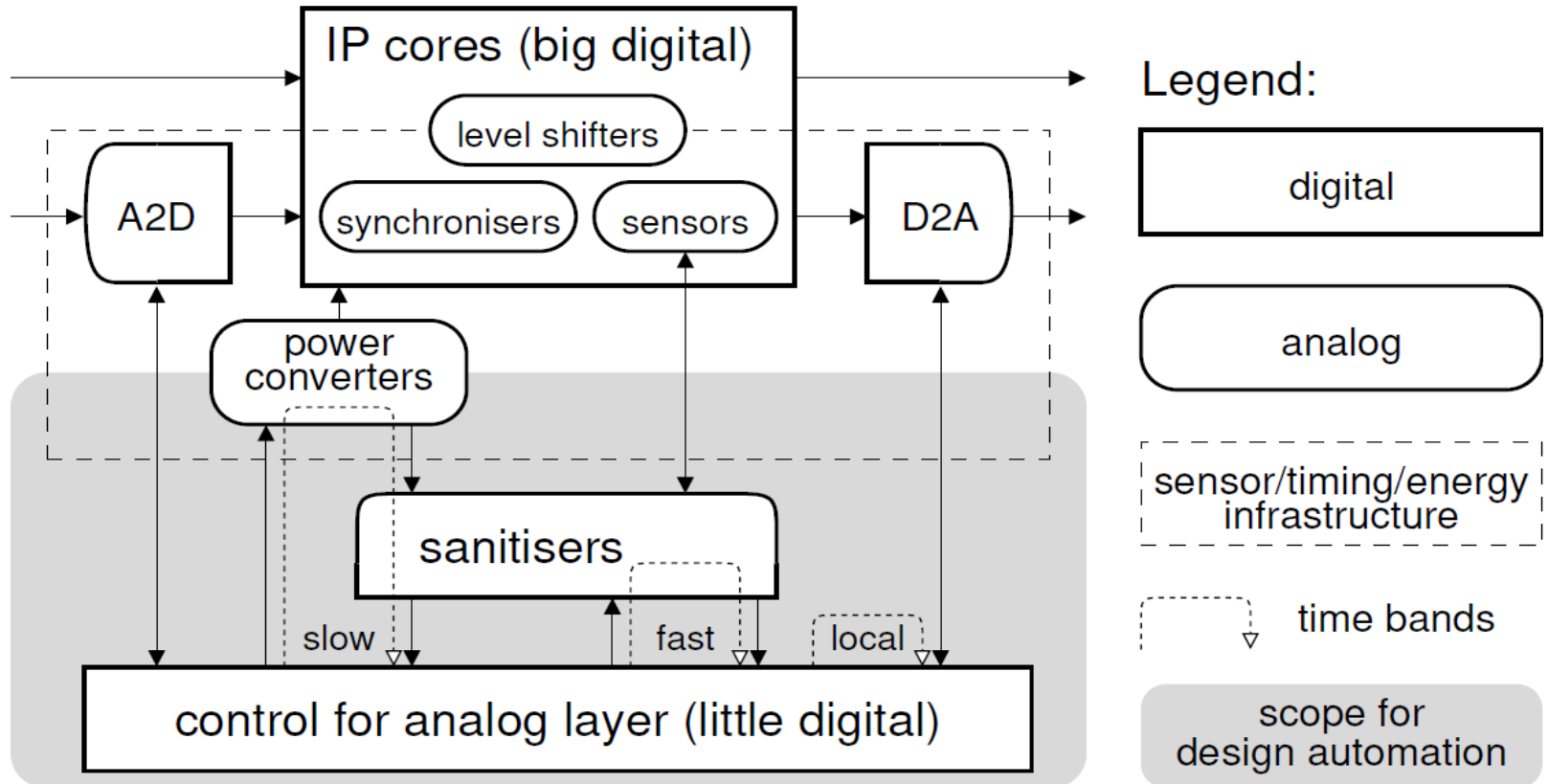
Motivation: power electronics context

- Efficient implementation of power converters is paramount
 - Extending battery life for mobile gadgets
 - Reducing energy bill for PCs and data centres (5% and 3% of global electricity production, respectively)
- Need for responsive and reliable control circuits – *little digital*
 - Millions of control decisions per second for years
 - A wrong decision may permanently damage the circuit (not as fuzzy as genetic circuits!)

Motivation: power electronics context

- Efficient implementation of power converters is paramount
 - Extended range of operation
 - Reduced losses (5% and 3% respectively)
- Need for Robustness, Low latency ...
 - Millions of years
 - A wrong circuit (not as fuzzy as genetic circuits!)

Emergence of little digital electronics



- Analog and digital electronics are becoming more intertwined
- Analog domain becomes complex and needs digital control

**And we have already done well
here, too!**

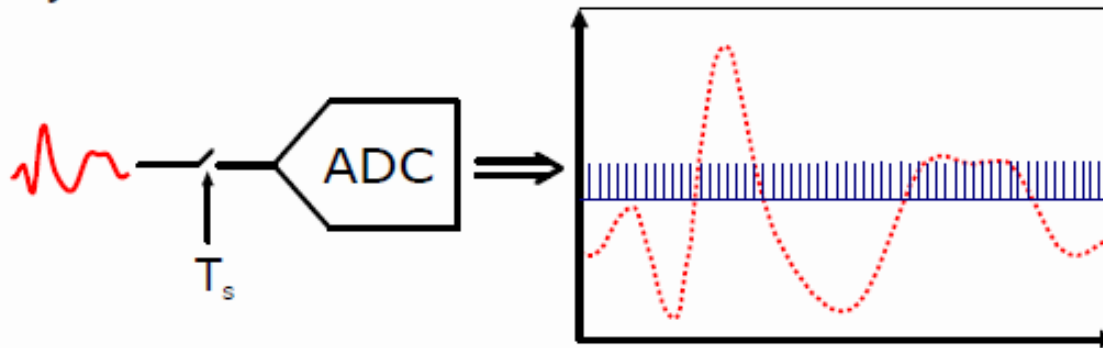
Async A-to-D Converters

Async-assisted sensors

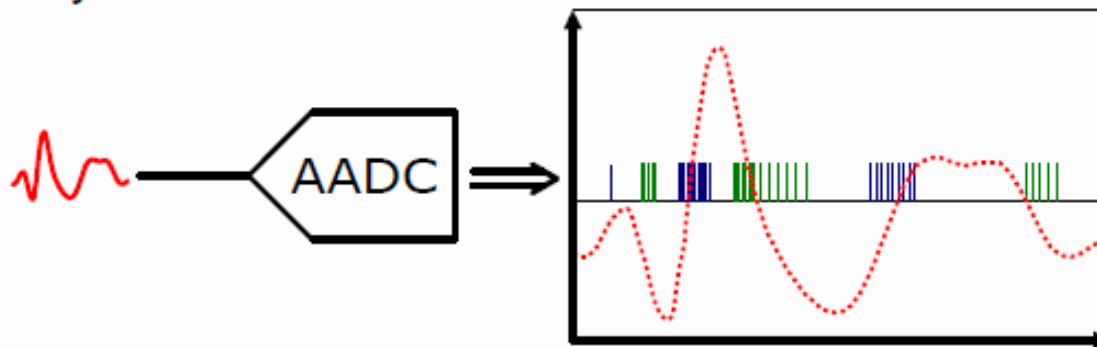
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Async ADC

- Synchronous

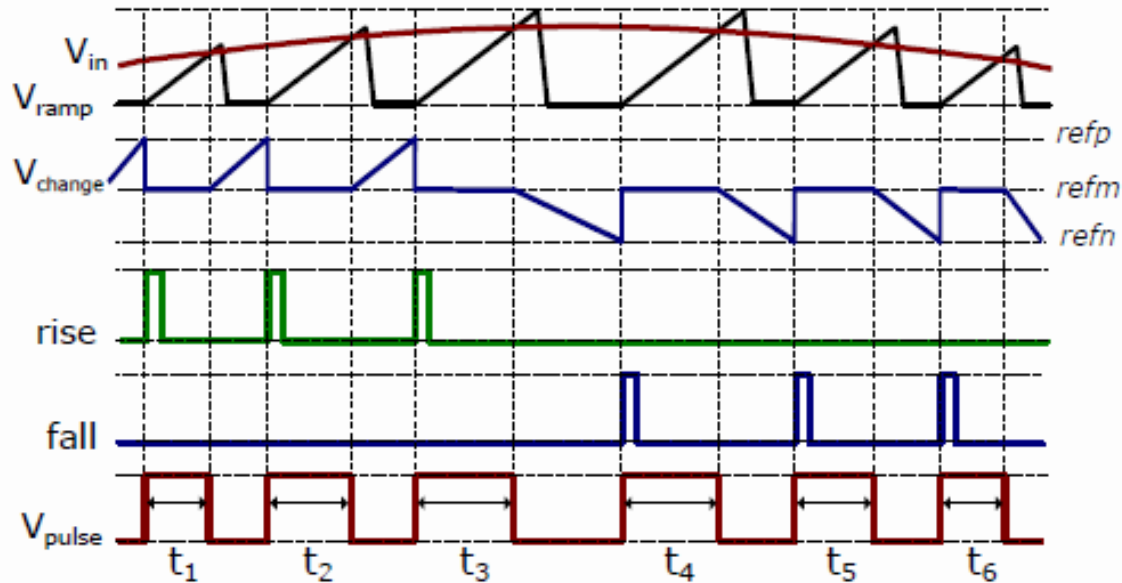
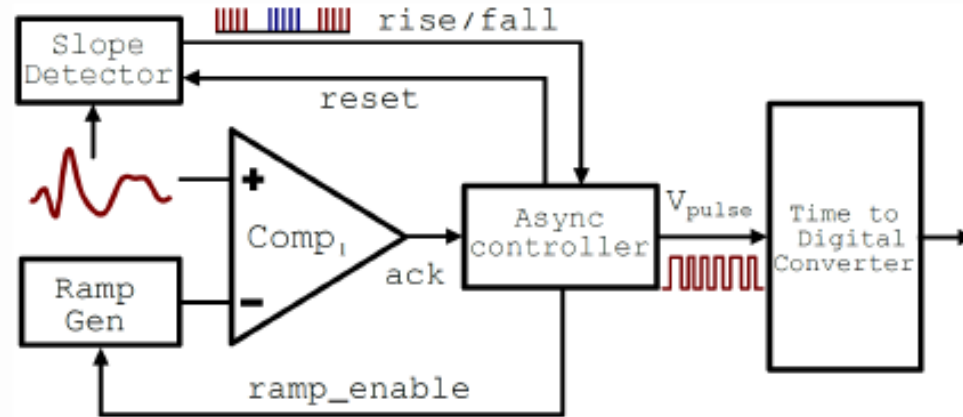


- Asynchronous



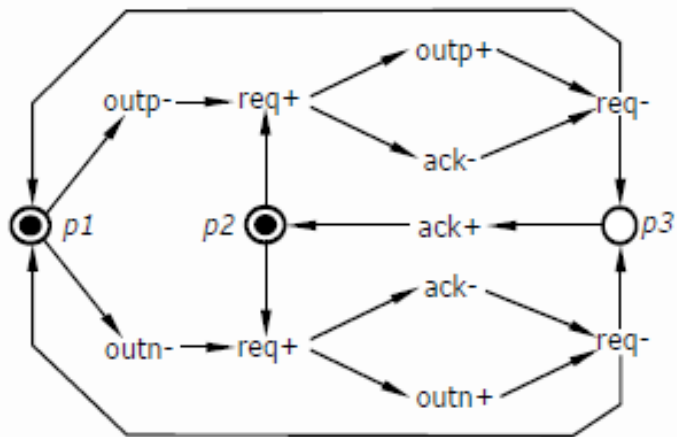
A. Ogwen, P. Degenaar, V. Khomenko and A. Yakovlev: "A fixed window level crossing ADC with activity dependent power dissipation", accepted for NEWCAS-2016.

ADC design

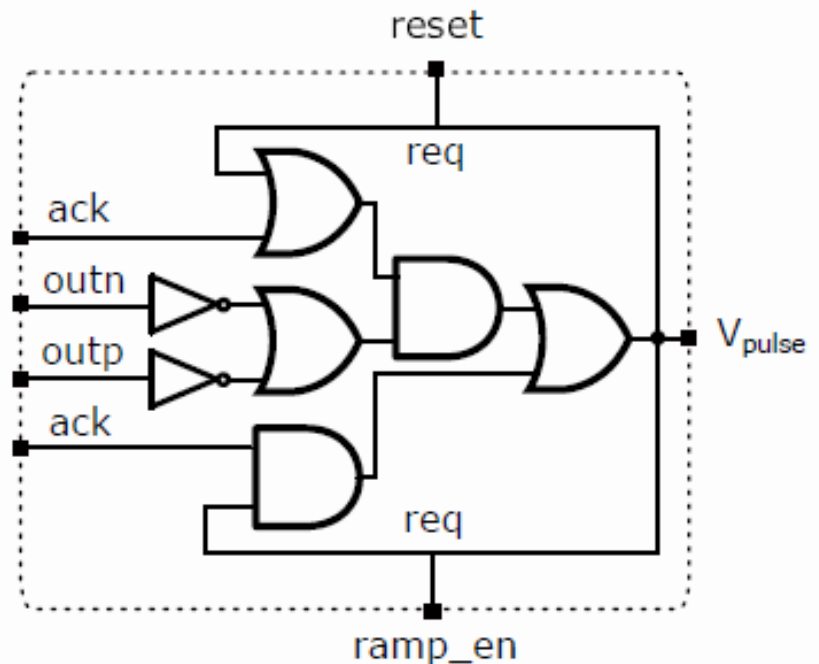


Asynchronous controller

- STG specification



- Speed-independent implementation



Reference-free voltage sensor

Energy harvesting
source



E_h

Storage
element

V_{dd}

Req

Control

Sampling
circuit

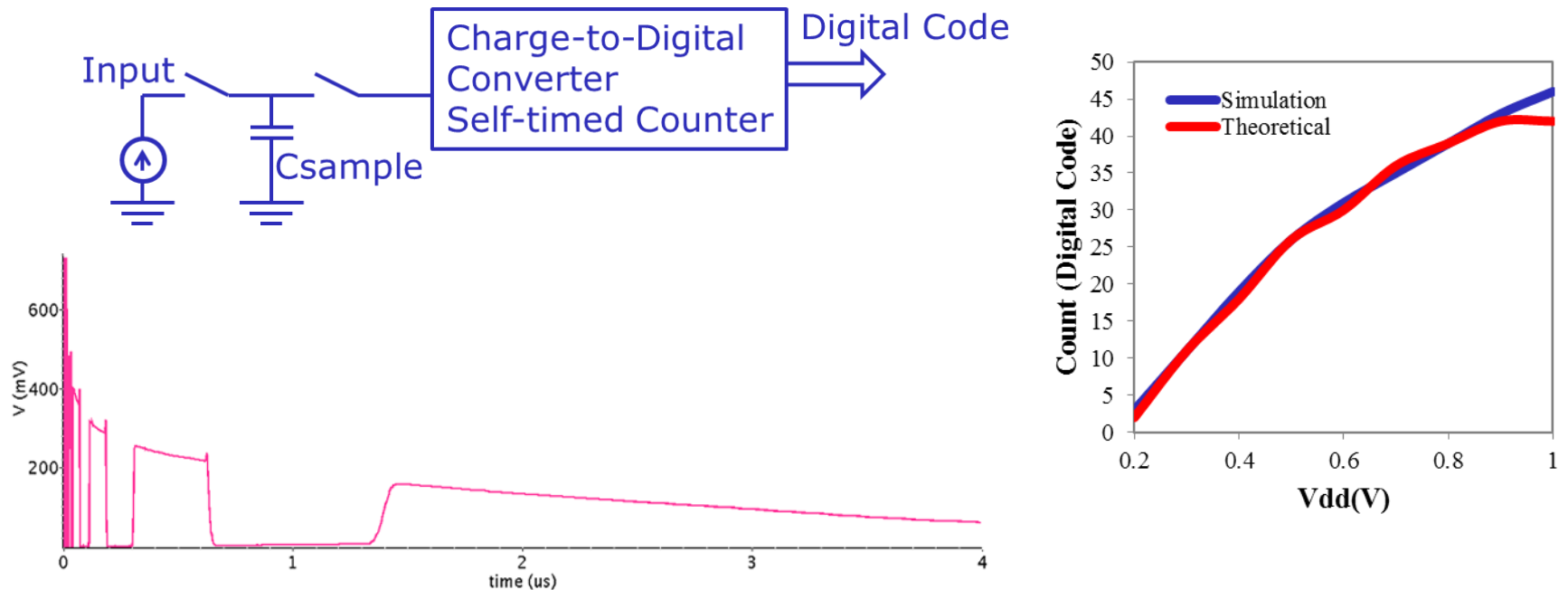
Self-timed
counter

8

Ack
Data

Reference-free voltage sensing

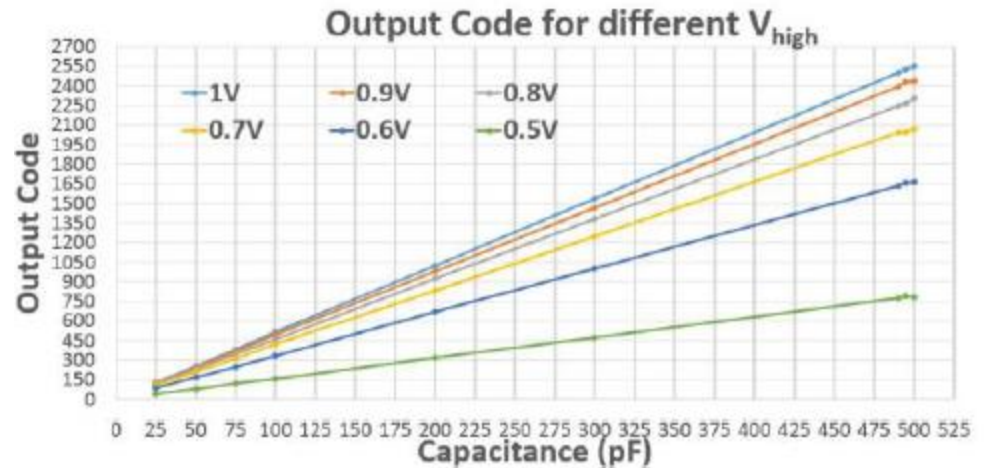
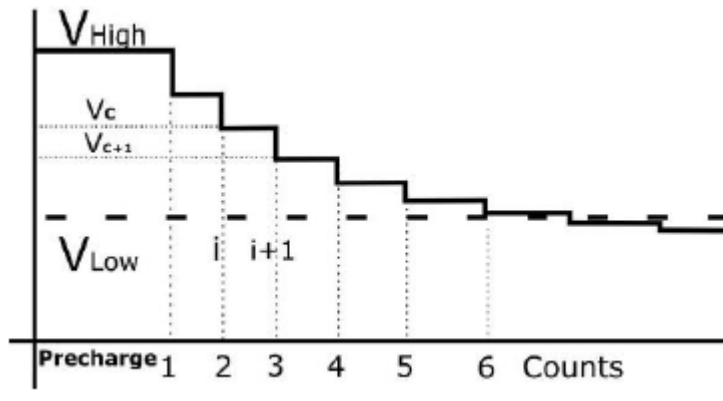
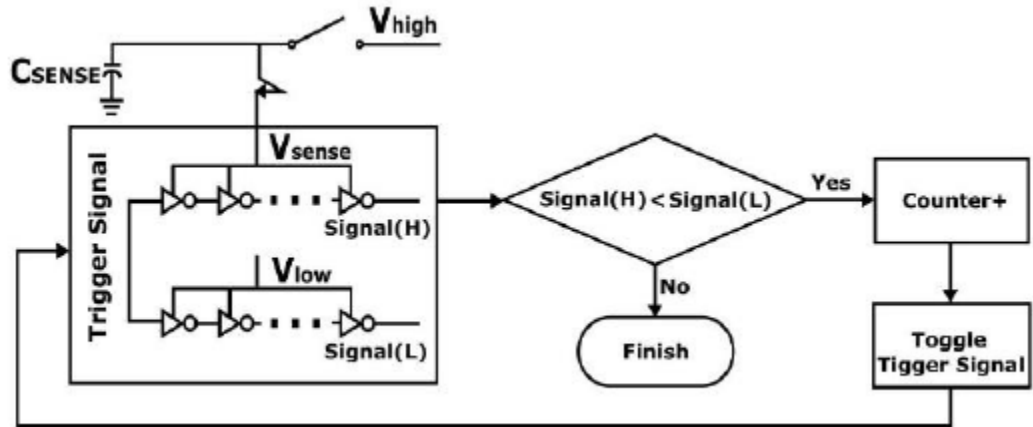
- Voltage sensor requiring only timing reference



Apparatus and method for voltage sensing, Newcastle University, GB Patent Number 2479156, 30 March 2010.

Sensors using asynchronous logic

Cap-to-digital Conversion (sensing)



Can we go further with Async?

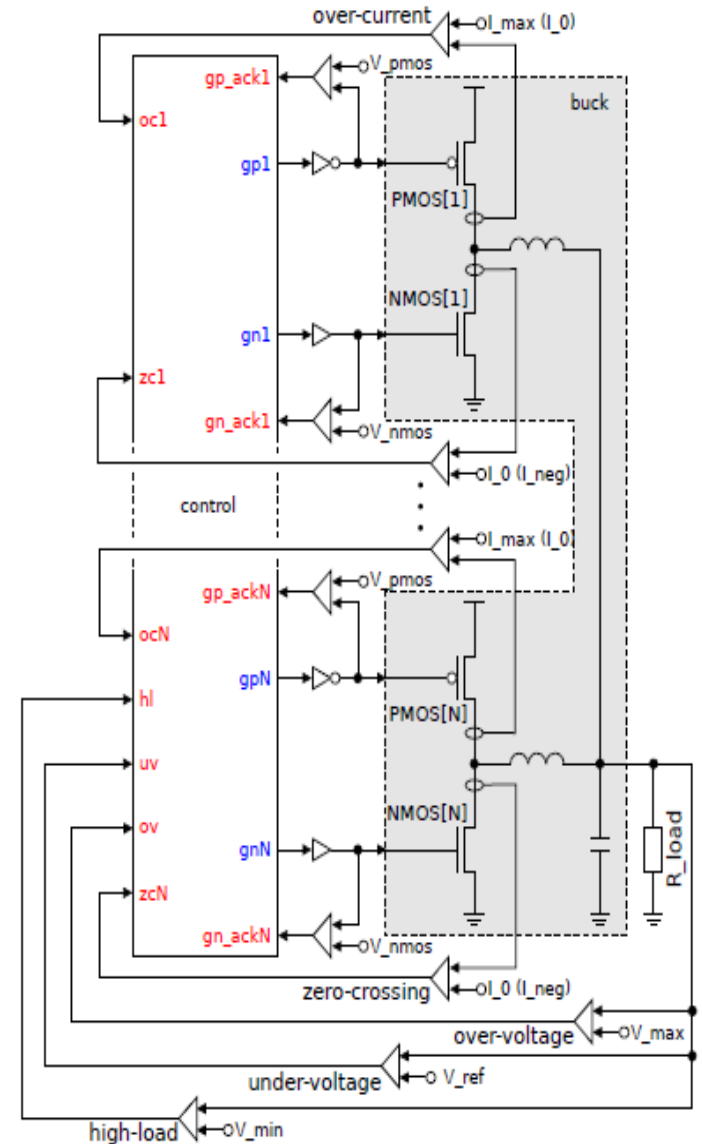
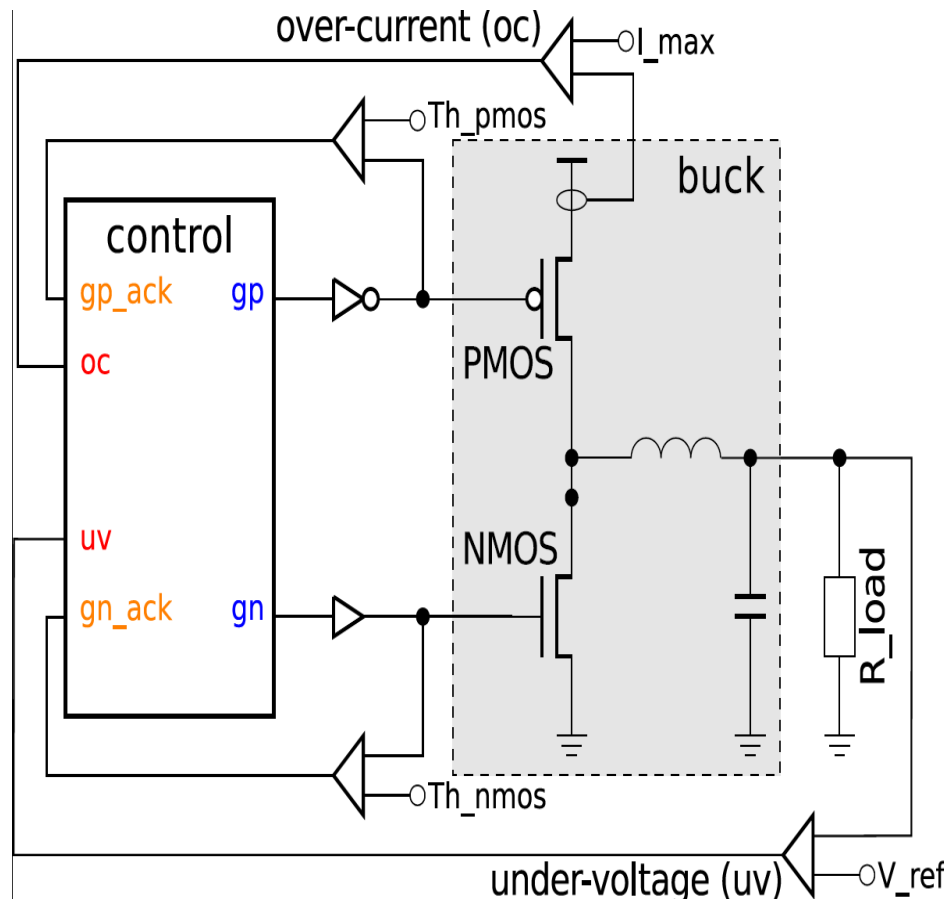
Where we can have a win-win situation!

To something bigger ... such as

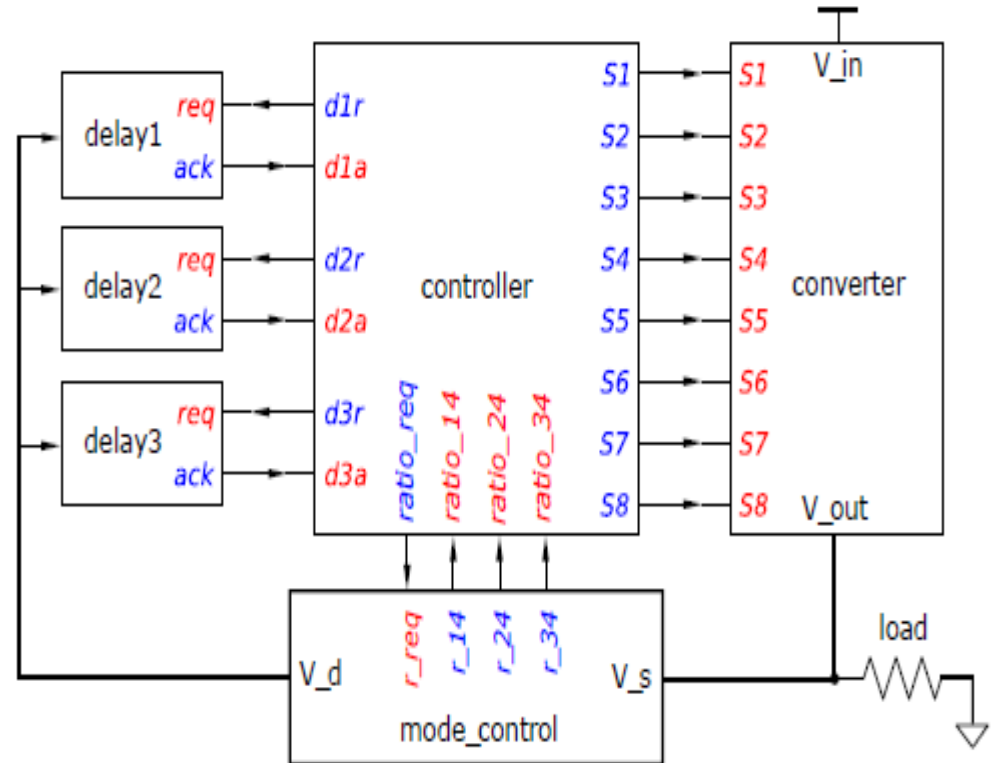
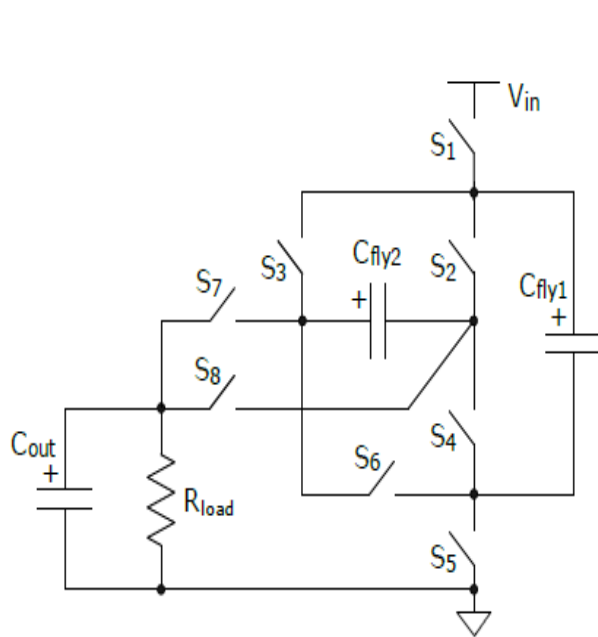
Power electronics

And show impact outside the 'usual' digital scope'

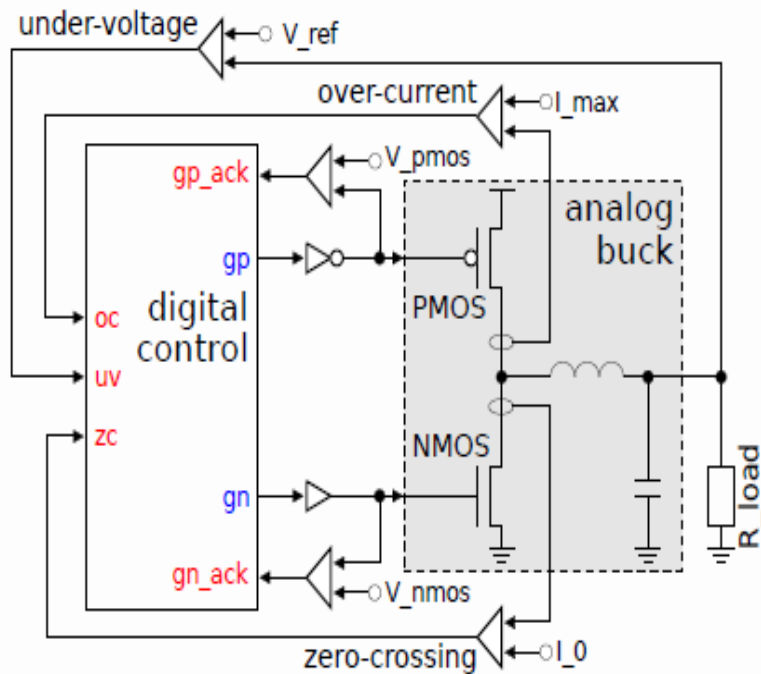
Example: Buck (DC-DC) converter control



Example: Switched Capacitor (DC-DC) Converter control

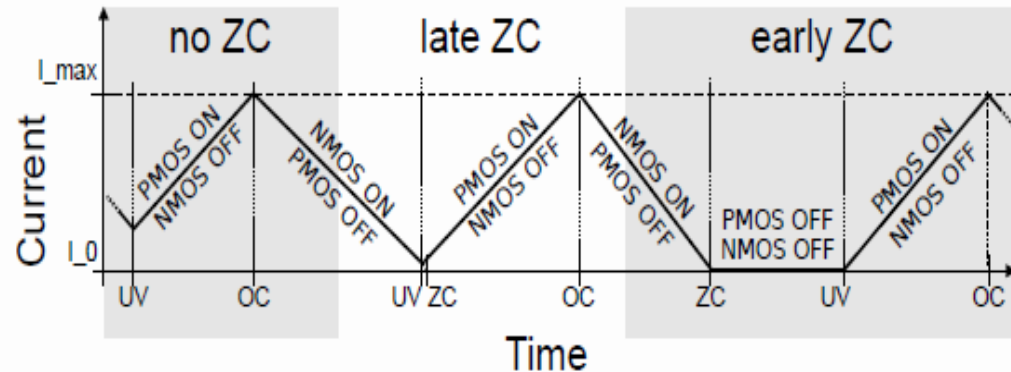


Example: Buck converter



Building asynchronous circuits in Analog-Mixed Signal context requires extending traditional assumptions about speed-independence ...

Phase diagram specification:



Buck conditions:

- under-voltage (UV)
- over-current (OC)
- zero-crossing (ZC)

Operating modes:

- no zero-crossing
- late zero-crossing
- early zero-crossing

Motivation: EDA support is a challenge

- **Poor EDA support**
 - Mostly supports flow from schematic capture; lacks flow from behavioural capture
 - Synthesis from behavioural (RTL) is optimized for data processing logic and supports only synchronous – *big digital*
 - *Manual* and *ad hoc* solutions are prone to errors and hard to verify (weeks of simulations)
- **Big challenge is EDA for asynchronous (hence our A4A project)**
- **What do the Industrial gurus say?**

Analog design in digital context is hard

- If digital parts don't use clock, they are normally designed by hand and require massive simulations:
 - E.g. analog designers cannot afford simulating power converters from start-up; Instead they force it into known state
 - More specifically: 50 us of Spectre simulation time takes approx. 10 hours using 8 CPU cores
 - Hence they can only verify cherry-picked corners of digital functionality
- (from Dialog Semiconductor, 2016)

Analog design in digital context is hard

- If digital parts don't use clock, they are normally designed by hand and require massive simulations:

- E.g. a ...ing power force ...ey
- More time ...on res
- Hence ...corners of di ... (2016)

Attack with the King:
Productivity!

Intel's advice on AMS Design

Intel's advice:

- Partition the Design to separate Analog and Digital
 - Give digital circuits to digital tools
 - Give analog circuits to analog tools
 - Do not pollute the hierarchy with logic gates or analog components!

(Source: Intel's talk about Holistic AMS design in Ultra-DSM at the May 2016 NMI event on AMS)

But, how?

We must use Behavioural capture and drive verification from behavioural domain!

View from Synopsys

Analysis and Debug

Data mining

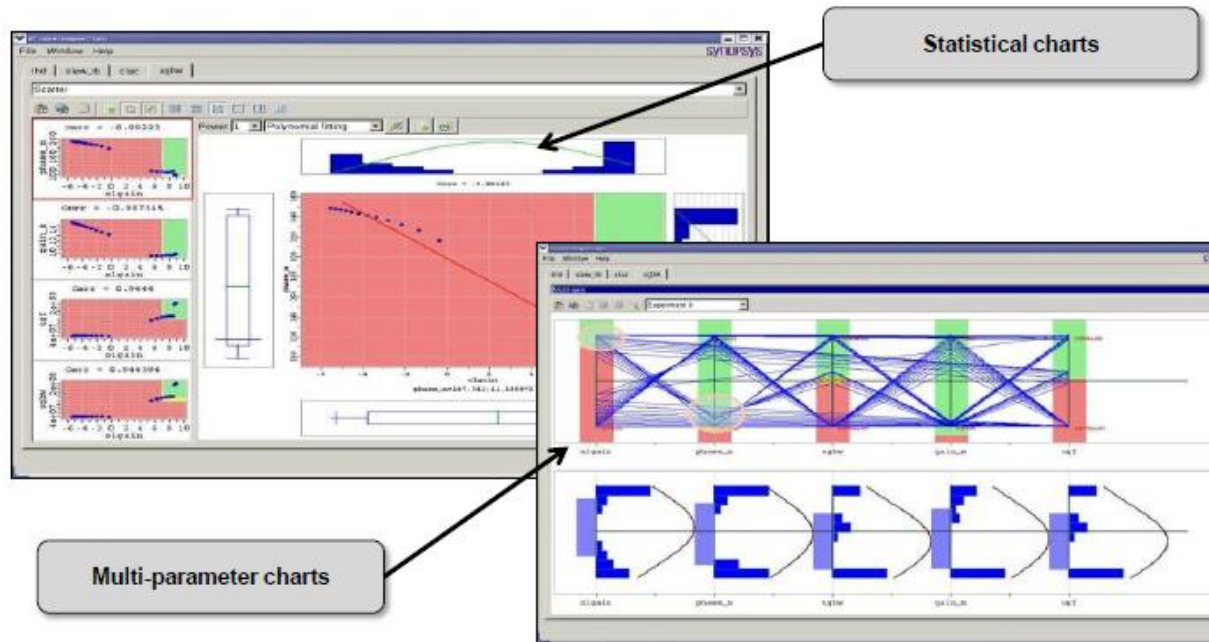
Design Data



Testbench Setup

Simulation
Management

Analysis &
Debug



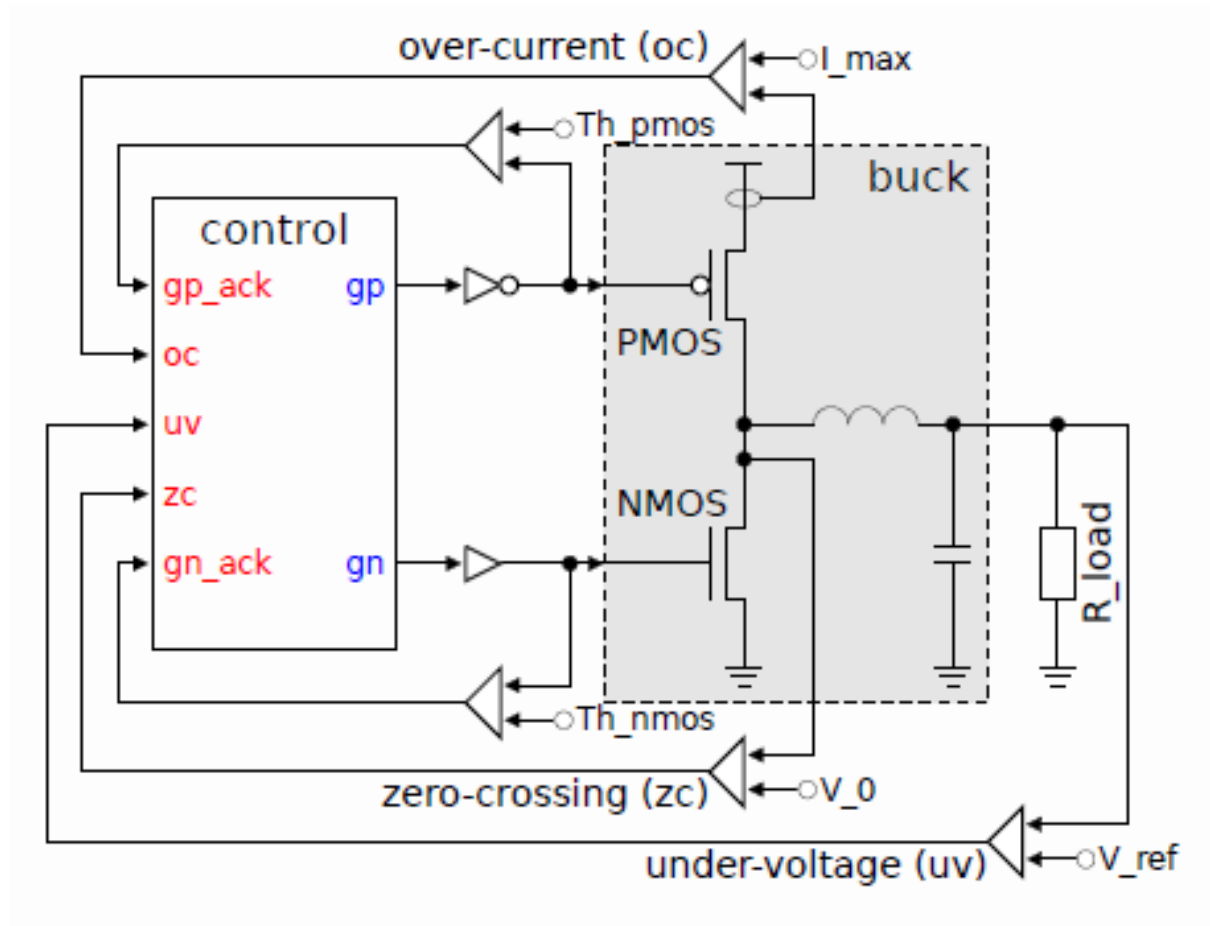
Comprehensive data mining with advanced statistical and multi-parameter charting

Source: Damian Roberts, AMS Workshop, RAL, April 2016

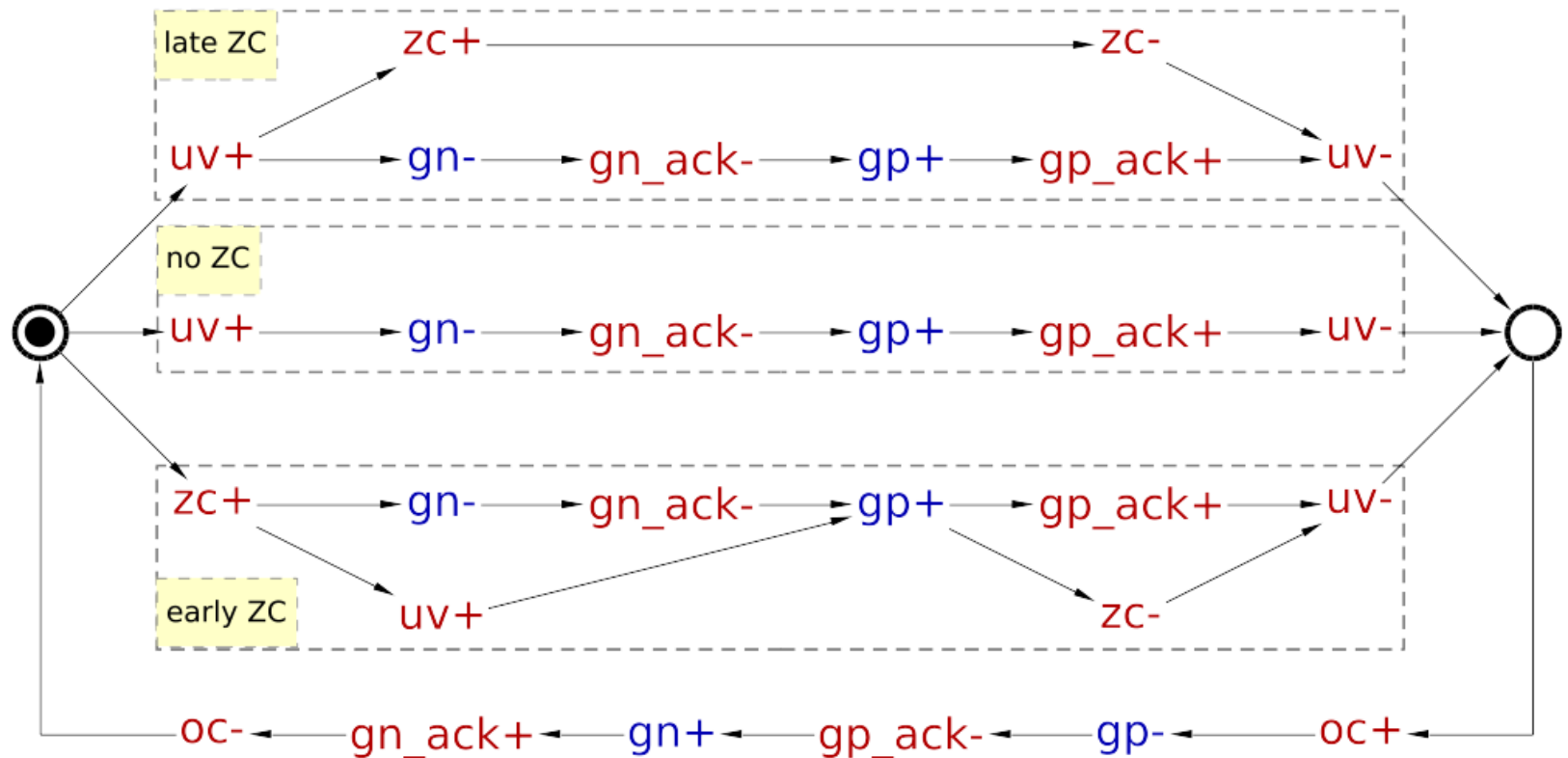
Towards Async Design for Analog

- **Asynchronous design offers many advantages for AMS control**
- **Challenges:**
 - **It requires behavioural capture and synthesis but commercial EDA tools don't support it**
 - **Verification of asynchronous designs as part of AMS**
 - **How to provide non-invasiveness with existing design practices – we need to work with SVA and SPICE simulation traces**

Buck example



STG Specification of buck controller



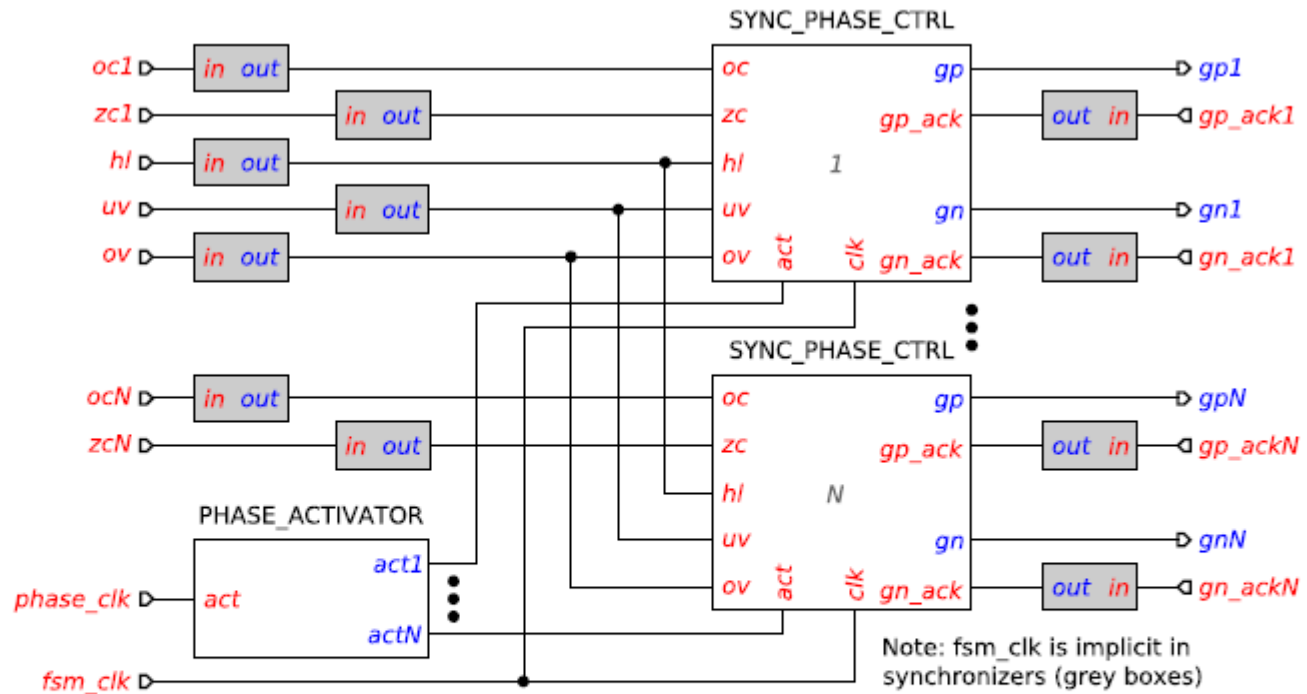
Synchronous design

- Two clocks: phase activation ($\sim 5\text{MHz}$) and sampling ($\sim 100\text{MHz}$)
 - 😊 Easy to design (RTL synthesis flow)
 - 😞 Response time is of the order of clock period
 - 😞 Power consumed even when idle
 - 😞 Non-negligible probability of a synchronisation failure
- Manual ad hoc design to alleviate the disadvantages
 - 😞 Verification by exhaustive simulation

Asynchronous design

- Event-driven control decisions
 - 😊 Prompt response (a delay of few gates)
 - 😊 No dynamic power consumption when the buck is inactive
 - 😊 Other well known advantages
 - 😞 Insufficient methodology and tool support
- Our goals
 - Formal specification of power control behaviour
 - Reuse of existing synthesis methods
 - Formal verification of the obtained circuits
 - Demonstrate new advantages for power regulation (power efficiency, smaller coils, ripple and transient response)

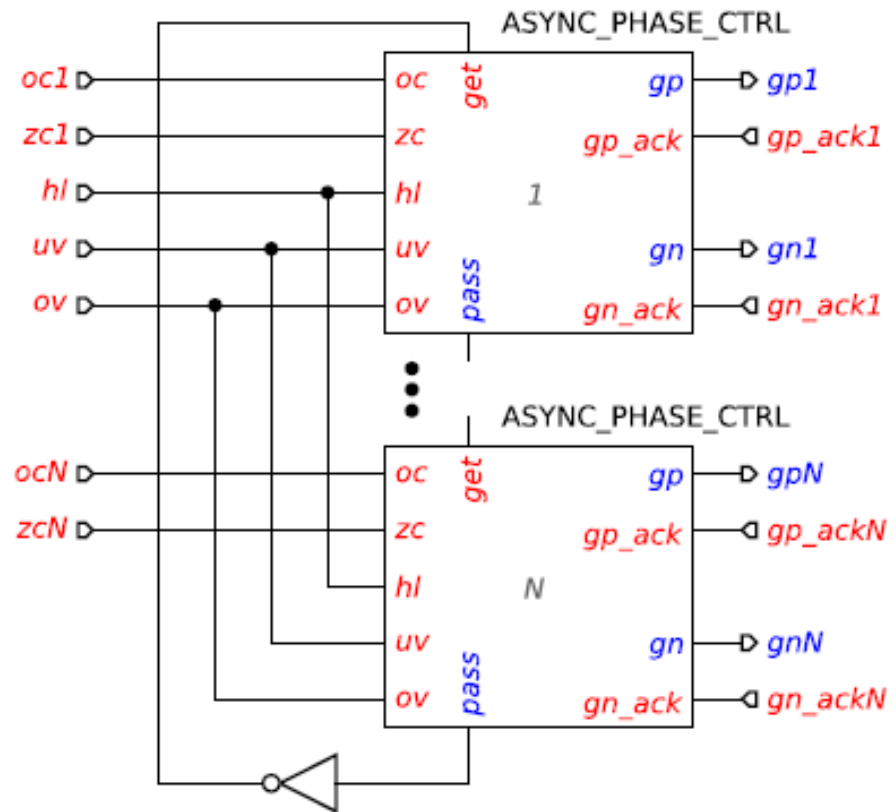
Multiphase Buck: Sync Control



- Two clocks: phase activation (slow) and sampling (fast)
- Need for multiple synchronizers (grey boxes) - latency & metastability
- Conventional RTL design flow

Multiphase Buck: Async Control

D. Sokolov, et al. DATE 2017

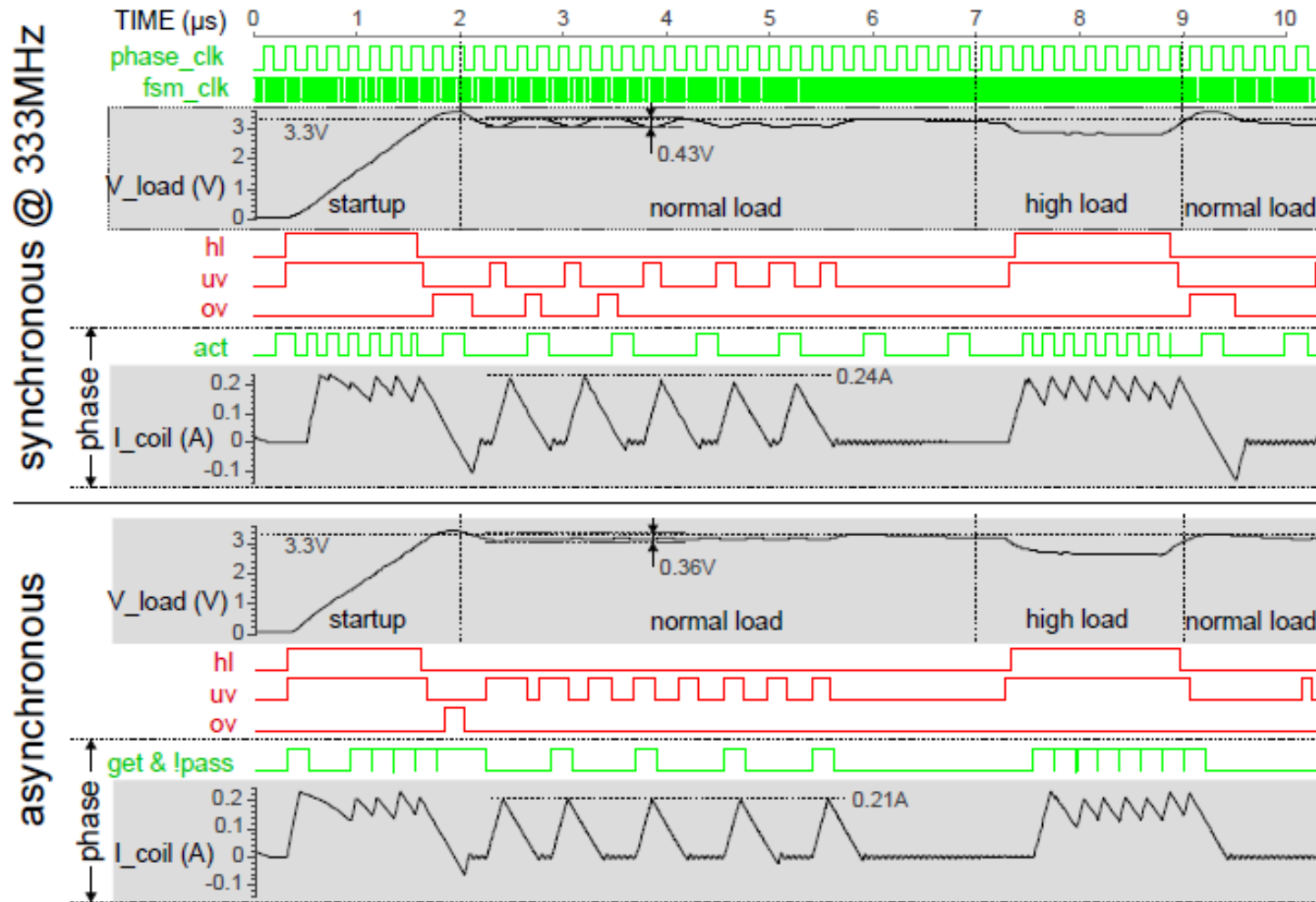


- Token ring architecture, no need for phase activation clock
- No need for synchronisers - all signals are asynchronous
- A4A design flow

Simulation results: Comparison

- Verilog-A model of the 3-phase buck
- Control implemented in TSMC 90nm
- AMS simulation in CADENCE NC-VERILOG
- Synchronous design
 - Phase activation clock – 5 MHz
 - Clocked FSM-based control – 100 MHz
 - Sampling and synchronisation
- Asynchronous design
 - Phase activation - token ring with 200 ns timer (= 5 MHz)
 - Event-driven control (input-output mode)
 - Waiting rather than sampling (A2A components)

Simulation results

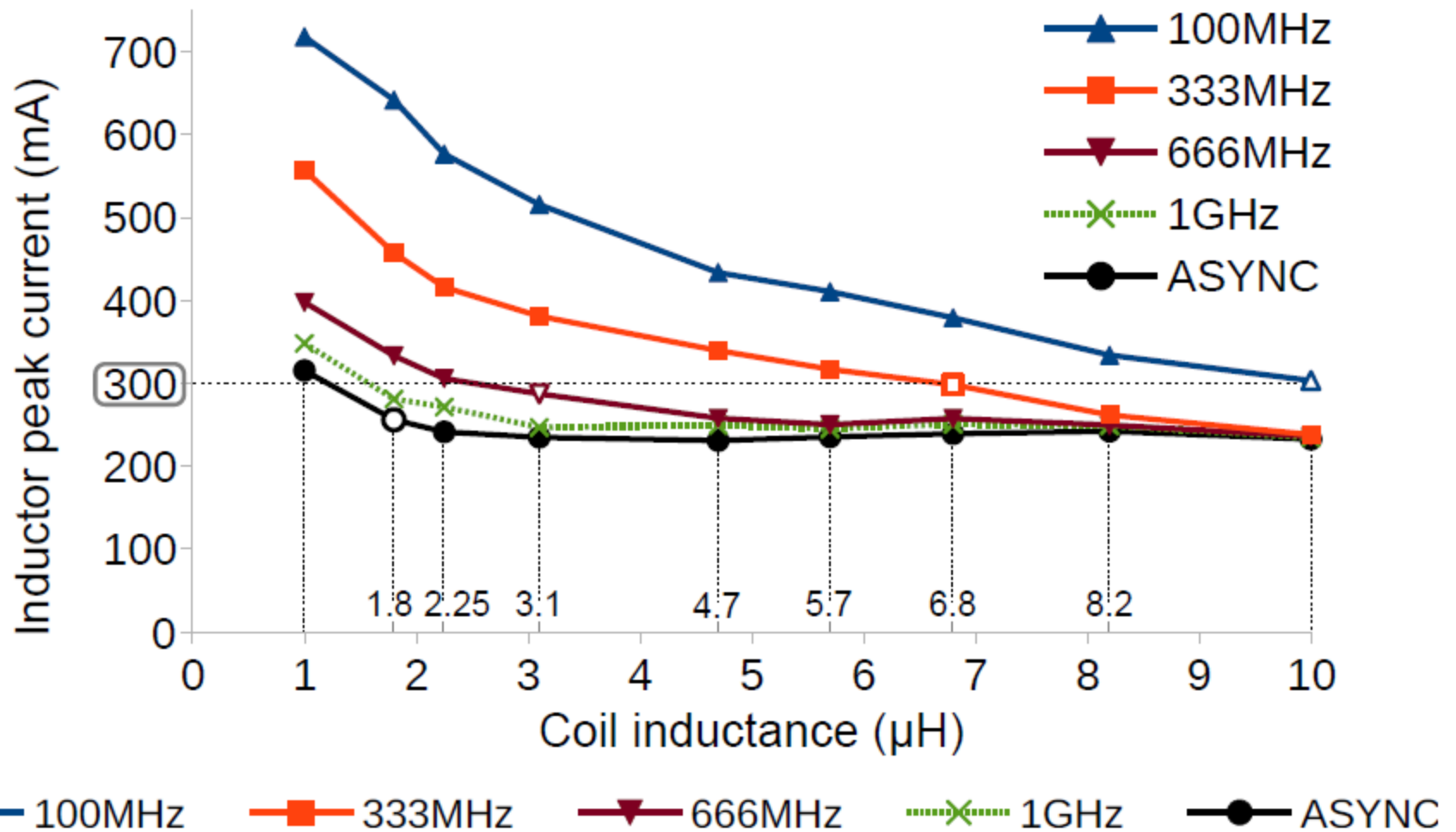


Reaction time

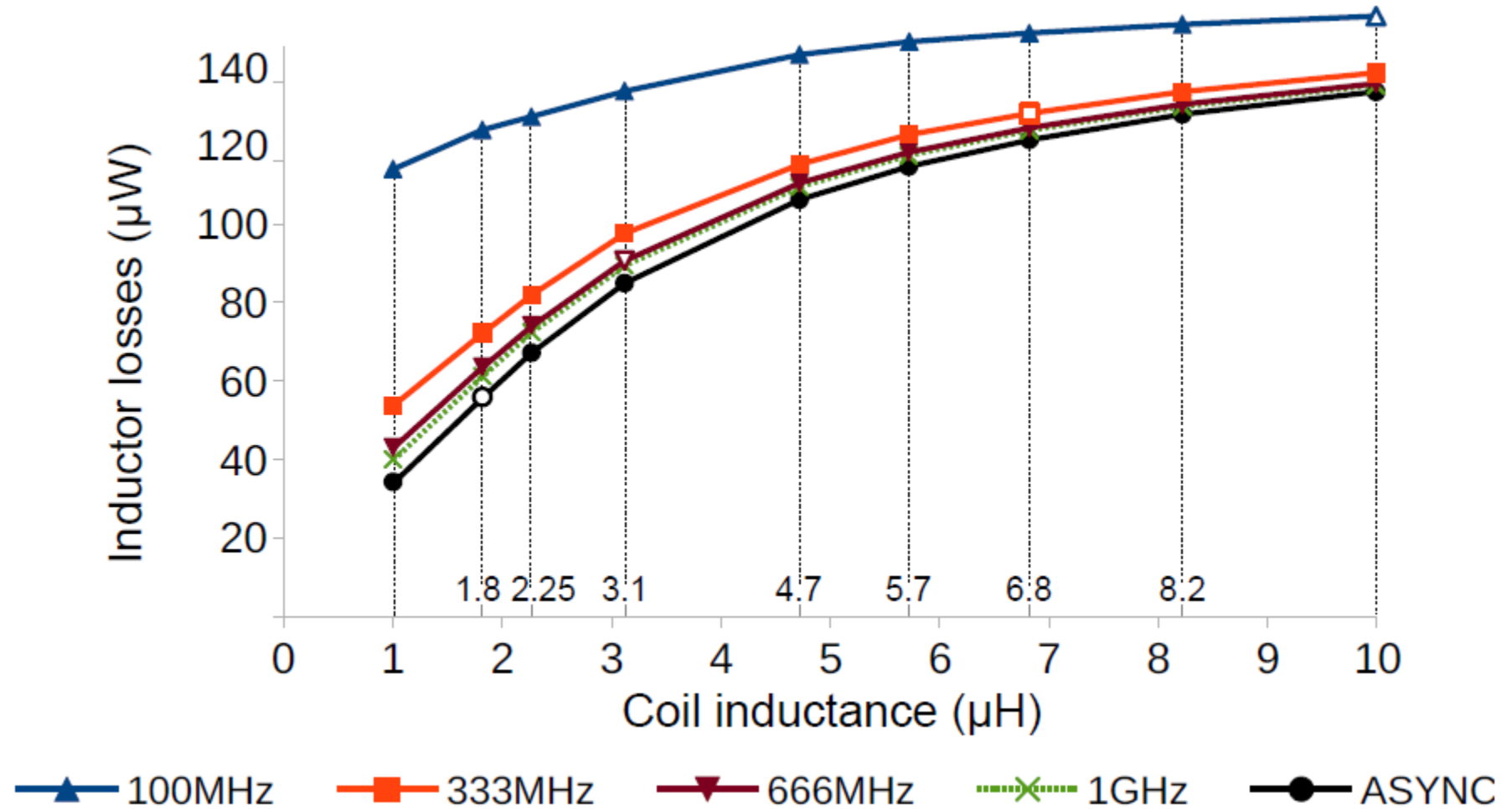
Buck controller	HL (ns)	UV (ns)	OV (ns)	OC (ns)	ZC (ns)
SYNC @ 100MHz	25.00	25.00	25.00	25.00	25.00
SYNC @ 333MHz	7.50	7.50	7.50	7.50	7.50
SYNC @ 666MHz	3.75	3.75	3.75	3.75	3.75
SYNC @ 1GHz	2.50	2.50	2.50	2.50	2.50
ASYNC	1.87	1.02	1.18	0.75	0.31
Improvement over 333MHz	4x	7x	6x	10x	24x

Synchronous buck controllers exhibit latency of 2.5 clock cycles.

Peak current



Inductor losses



Design Results

Design flow is automated to large extent

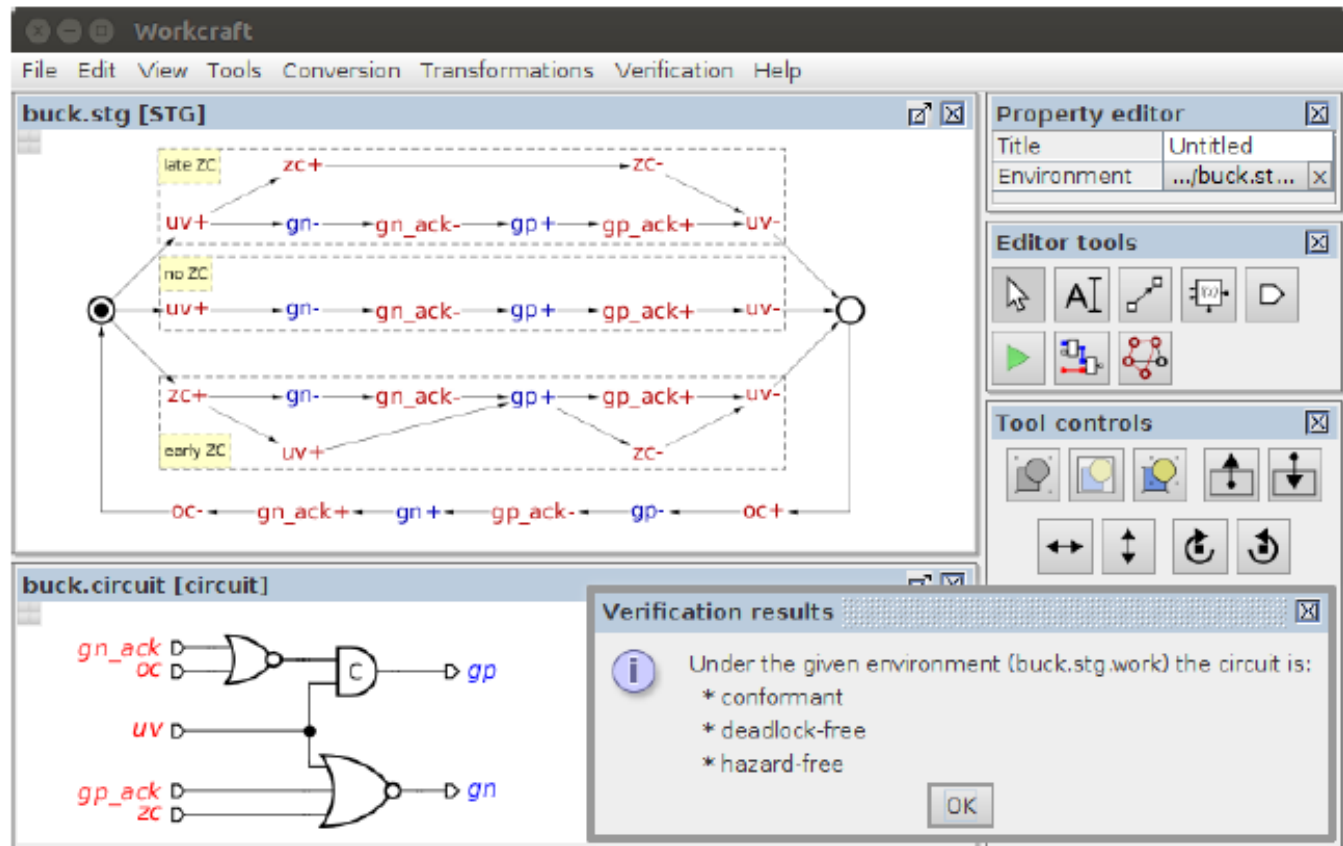
- Library of A2A components
- Automatic logic synthesis
- Formal verification at the STG and circuit levels

Analog-2-Async (A2A):
Wait, WaitX, Sample...

Benefits of asynchronous multiphase buck controller

- Reliable, no synchronization failures
- Quick response time (few gate delays)
- Reaction time can be traded off for smaller coils
- Lower voltage ripple and peak current

Logic synthesis and formal verification of asynchronous circuits.



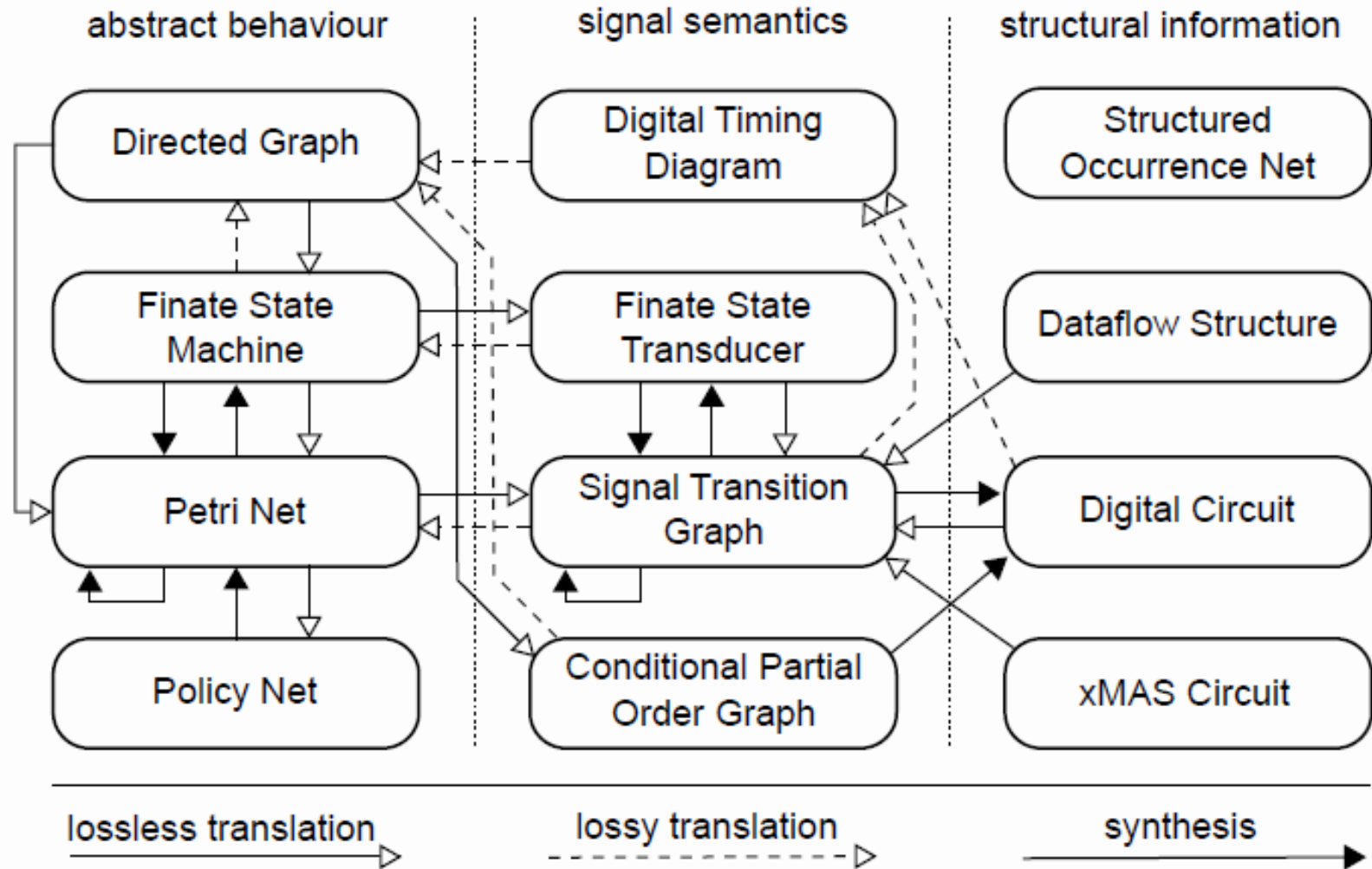
What is Workcraft?

- Framework for interpreted graph models
 - Interoperability between different abstraction levels
 - Consistency for users; convenience for developers
- Elaborate graphical user interface
 - Visual editing, analysis, and simulation
 - Easy access to common operations
 - Possibility to script specialised actions
- Interface to back-end tools for synthesis and verification
 - Reuse of established theory and tools (PETRIFY, MPSAT, PUNF)
 - Command log for debugging and scripting

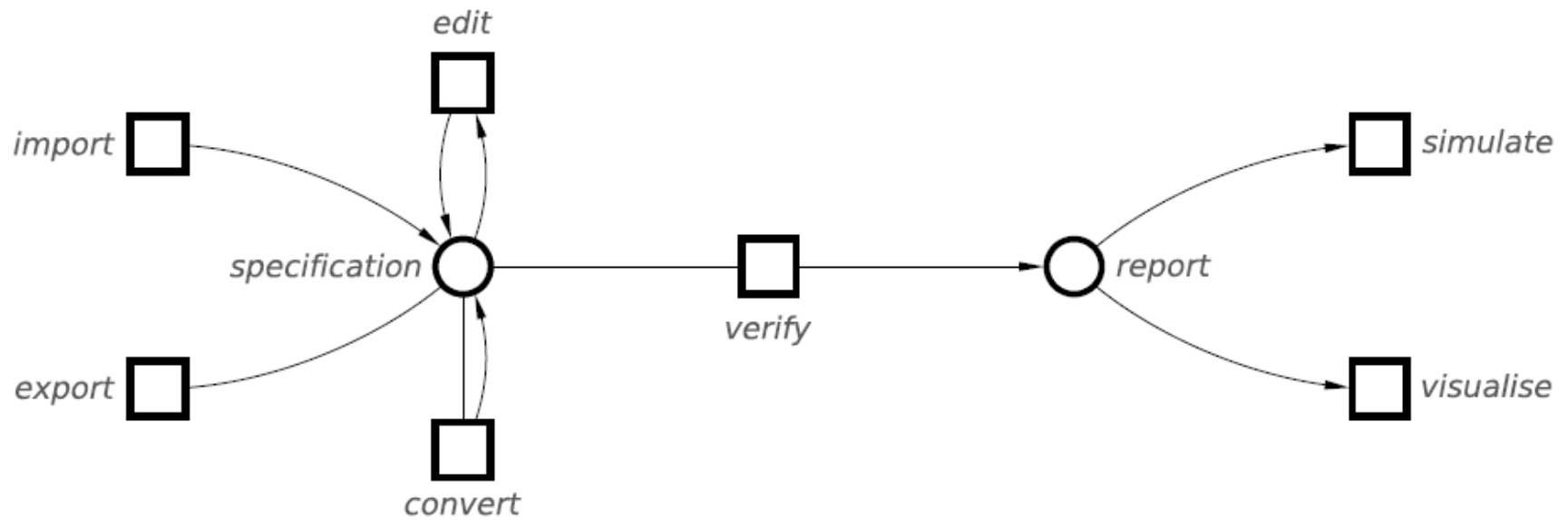
Why to use Wokcraft?

- Availability
 - Open-source front-end and plugins
 - Permissive freeware licenses for back-end tools
 - Frequent releases (4-6 per year)
 - Specialised tutorials and online training materials
- Extendibility
 - Plugins for new formalisms
 - Import, export and converter plugins
 - Interface to back-end tools
- Usability
 - Elaborated GUI developed with much user feedback
- Portability
 - Distributions for Windows, Linux, and OS X

Supported graph models



Workcraft Design Flow



- Import: ASTG, Verilog
- Export: ASTG, Verilog, SVG/Dot/PDF/EPS
- Convert: synthesis or translation
- Verify: reachability analysis

Design flow: asynchronous circuits

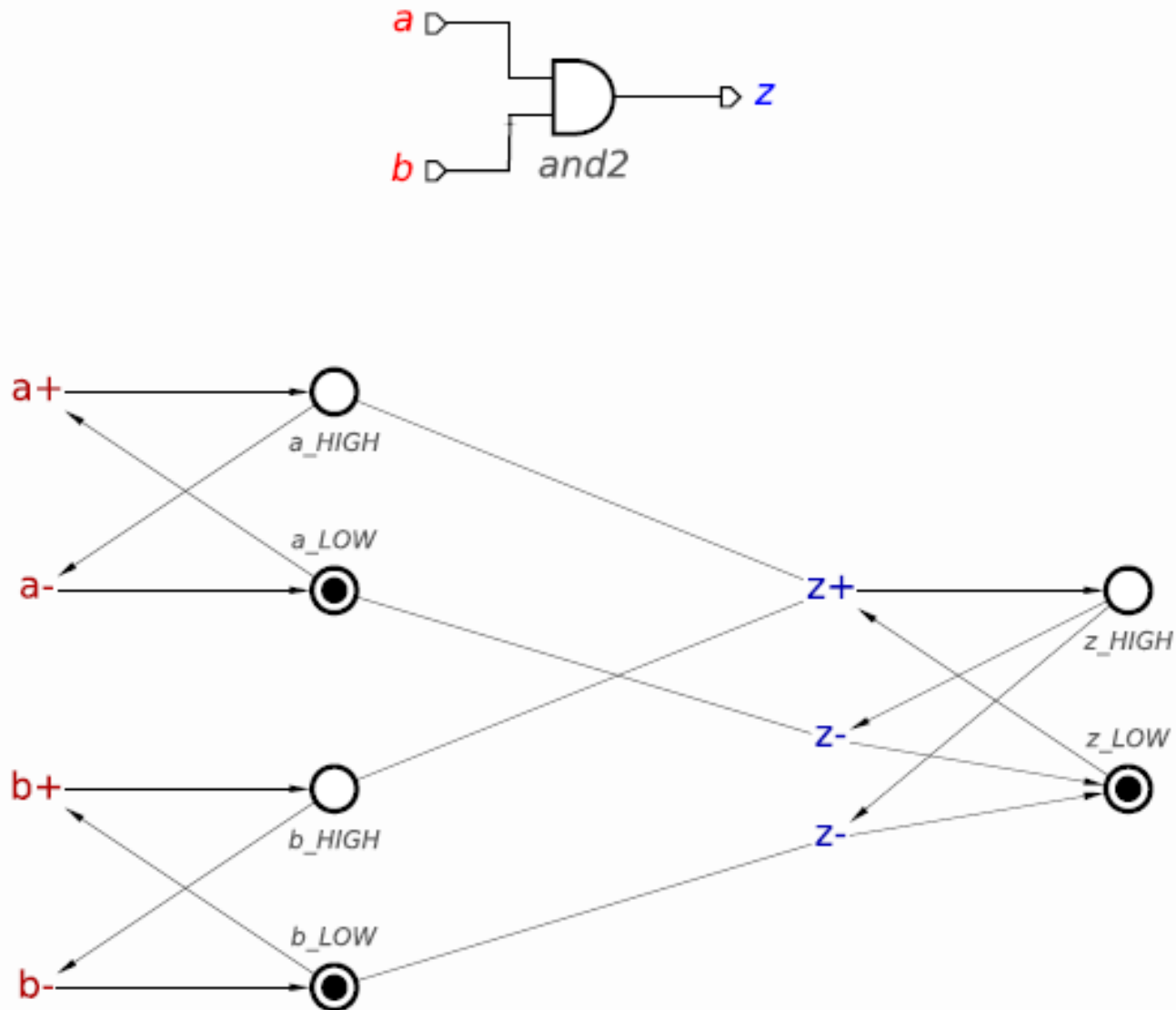
1. Specification of desired circuit behaviour with an STG model
2. Verification of the STG model
 - (a) Standard implementability properties:
consistency, deadlock freeness, output persistency
 - (b) Design-specific custom properties
3. Resolution of complete state coding (CSC) conflicts
4. Circuit synthesis in one of the supported design styles
5. Manual tweaking and optimisation of the circuit
6. Verification of circuit against the initial specification
 - (a) Synthesis tools are complicated and may have bugs
 - (b) Manual editing is error-prone
7. Exporting the circuit as a Verilog netlist for conventional EDA backend

What is hidden from the user

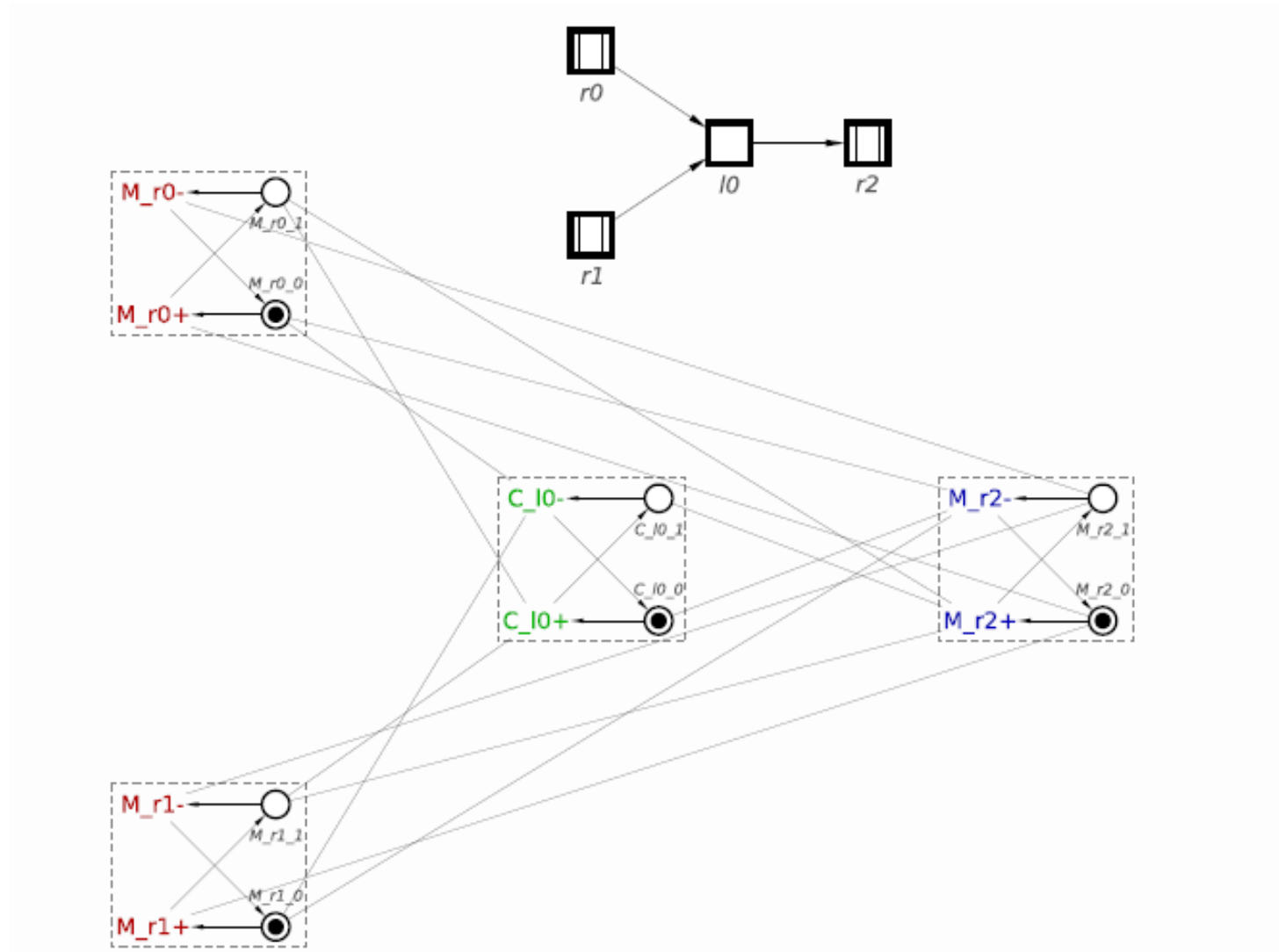
Verification that the circuit conforms to its specification

1. Circuit is converted to an equivalent STG – circuit STG
2. Internal signal transitions in the environment STG (contract between the circuit and its environment) are replaced by dummies
3. Circuit STG and environment STG are composed by PCOMP back-end
4. Conformation property is expressed in REACH language
5. Composed STG is unfolded by calling PUNF back-end
6. Unfolding prefix and REACH expression are passed to MPSAT back-end
7. Verification results are parsed by the front-end
8. Violation trace is projected to the circuit for simulation and debugging

Circuit Petri net as assembly language



Circuit Petri nets: data flow pipelines

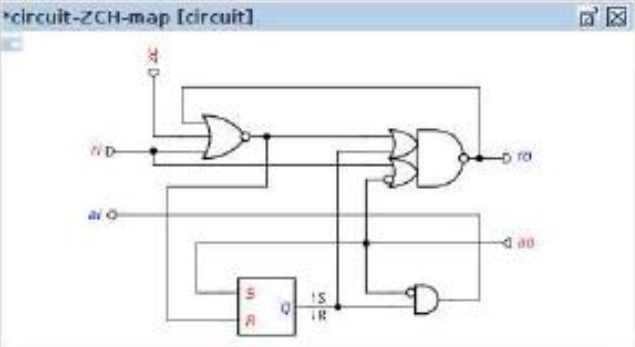


Workcraft: basic screenshot

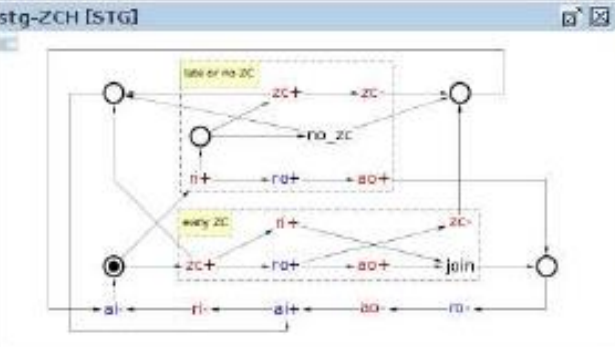
Workcraft

File Edit View Tools Help

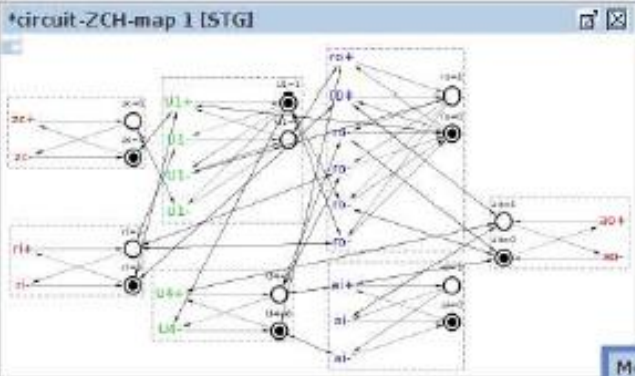
*circuit-ZCH-map [circuit]



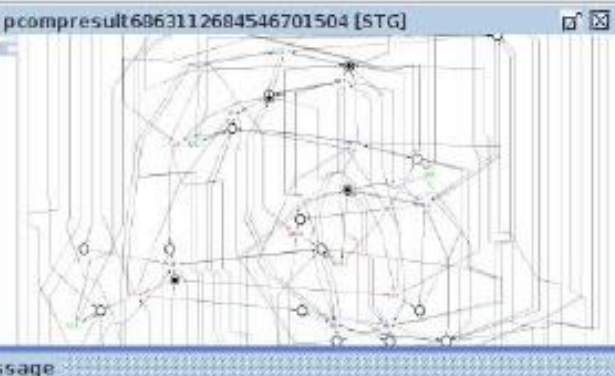
stg-ZCH [STG]



*circuit-ZCH-map 1 [STG]



pcompresult6863112684546701504 [STG]



Property editor [Model]

Environment... ./stg-ZCH...

Tool controls

Editor tools

workspace

workspace

External

- circuit-ZCH-map 1.work
- circuit-ZCH-map.work *
- stg-ZCH.work
- pcompresult68631126845

Output Problems Javascript Tasks

```
INORDER = ao ri zc ai ro csc0;  
OUTORDER = [ai] [ro] [csc0];  
[ai] = csc0 ao'; # gate and2_1:combinational  
[1] = ri' zc' ro'; # gate nor3:combinational  
#PRAGMA: zero delay  
[2] = ao'; # gate inv:combinational  
[ro] = [1]' csc0' + [2]' ri'; # gate and22:combinational  
[csc0] = csc0 [1]' + ao'; # gate sr_nor:asynch  
  
# Set/reset pins: reset(ro)  
Exporting model "Untitled" to file "/tmp/workcraft-circuit-ZCH-map-8245652389417126911/dev.g".
```

Message

Under the given environment (stg-ZCH.work) the circuit is:

- * conformant
- * deadlock-free
- * hazard-free

OK

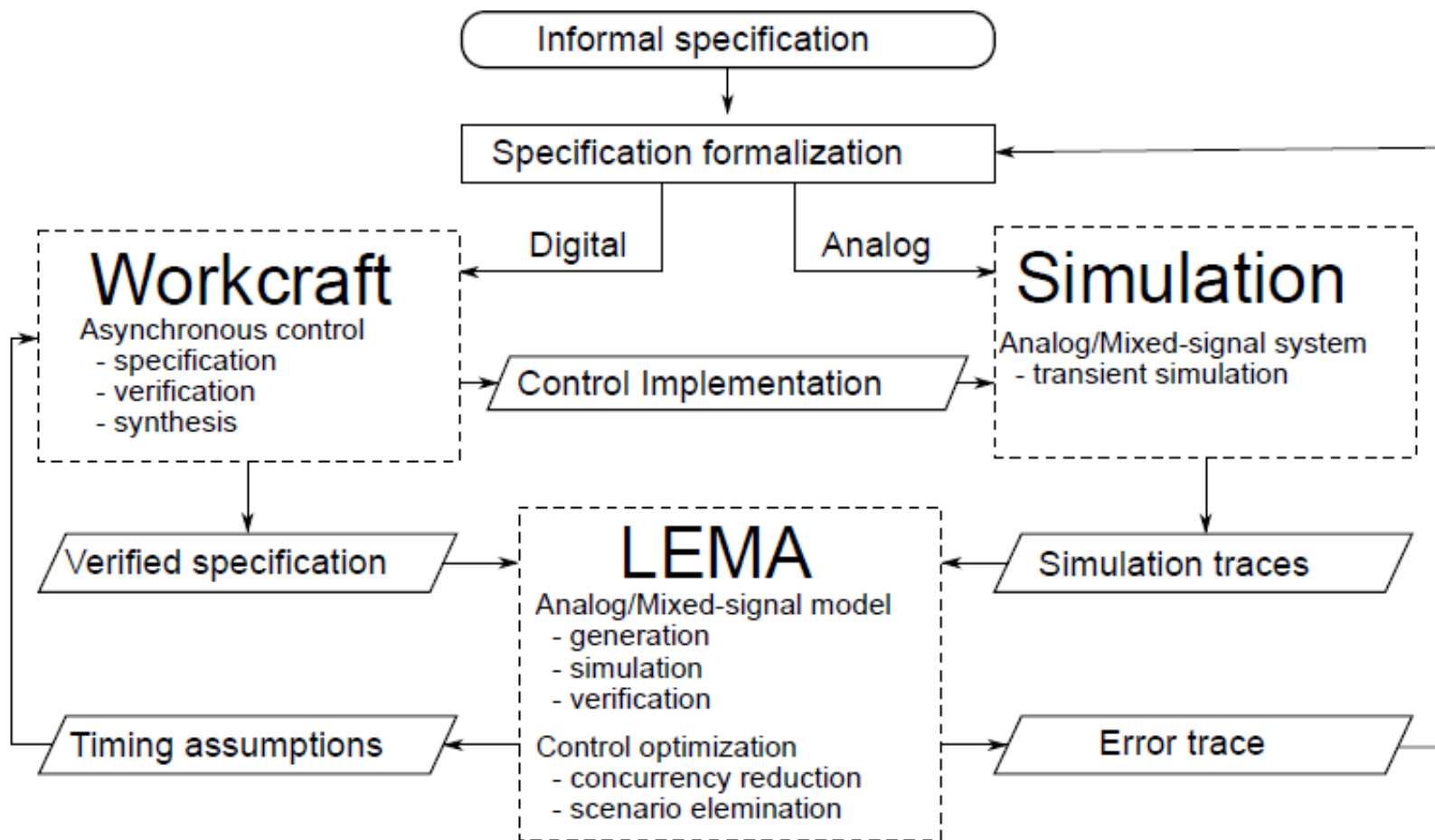
Specifics of Async Design for Bucks

- Needs to be to a large extent monolithic
- Has inputs that need to be sanitised
- Can have lots of timing assumptions for bounded delay implementation where solving coding and TM problems can be an issue
- I/O response times (constrained or optimised) drive the design and sign-off
- Different types of (de)compositions needed rather than (or not just) handshake ones

Future: Analog-Async Codesign!



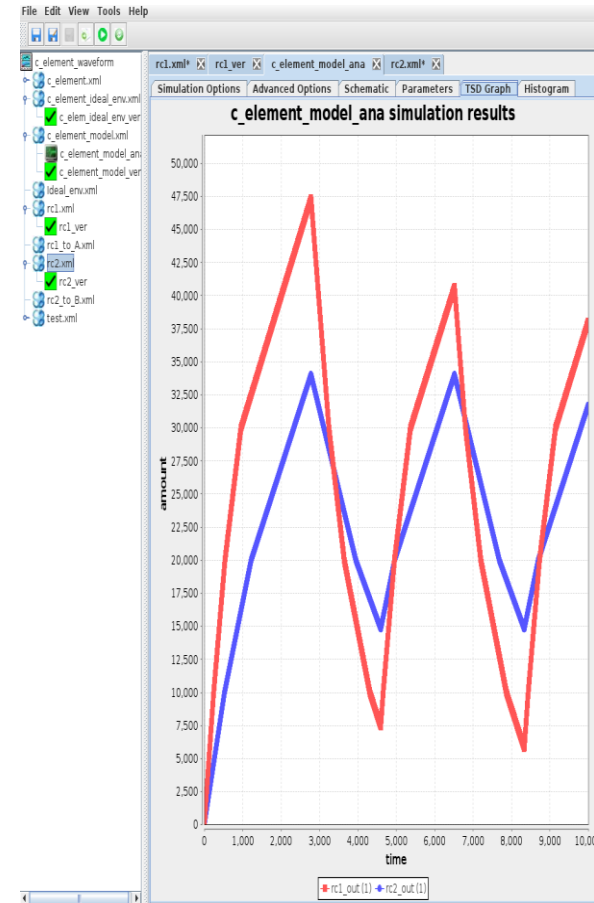
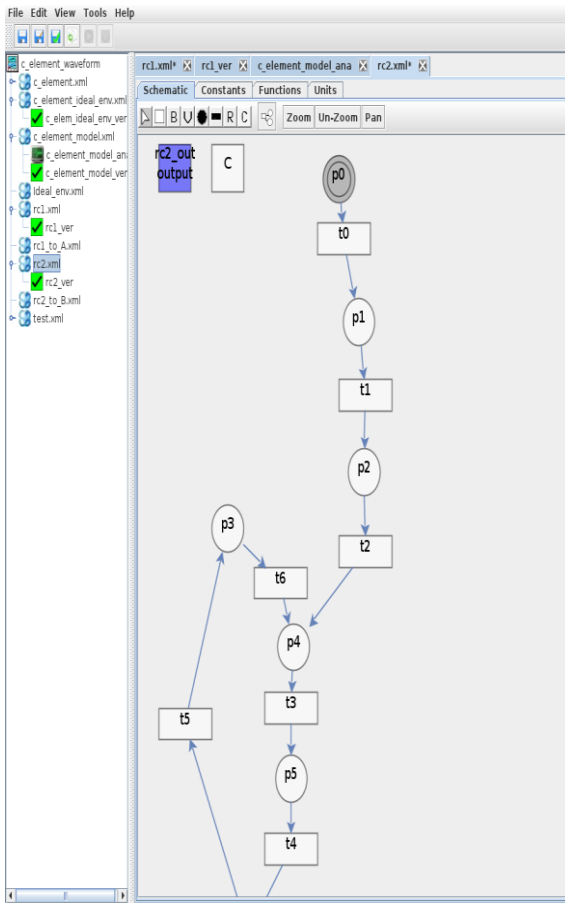
LEMA-Workcraft Codesign Flow



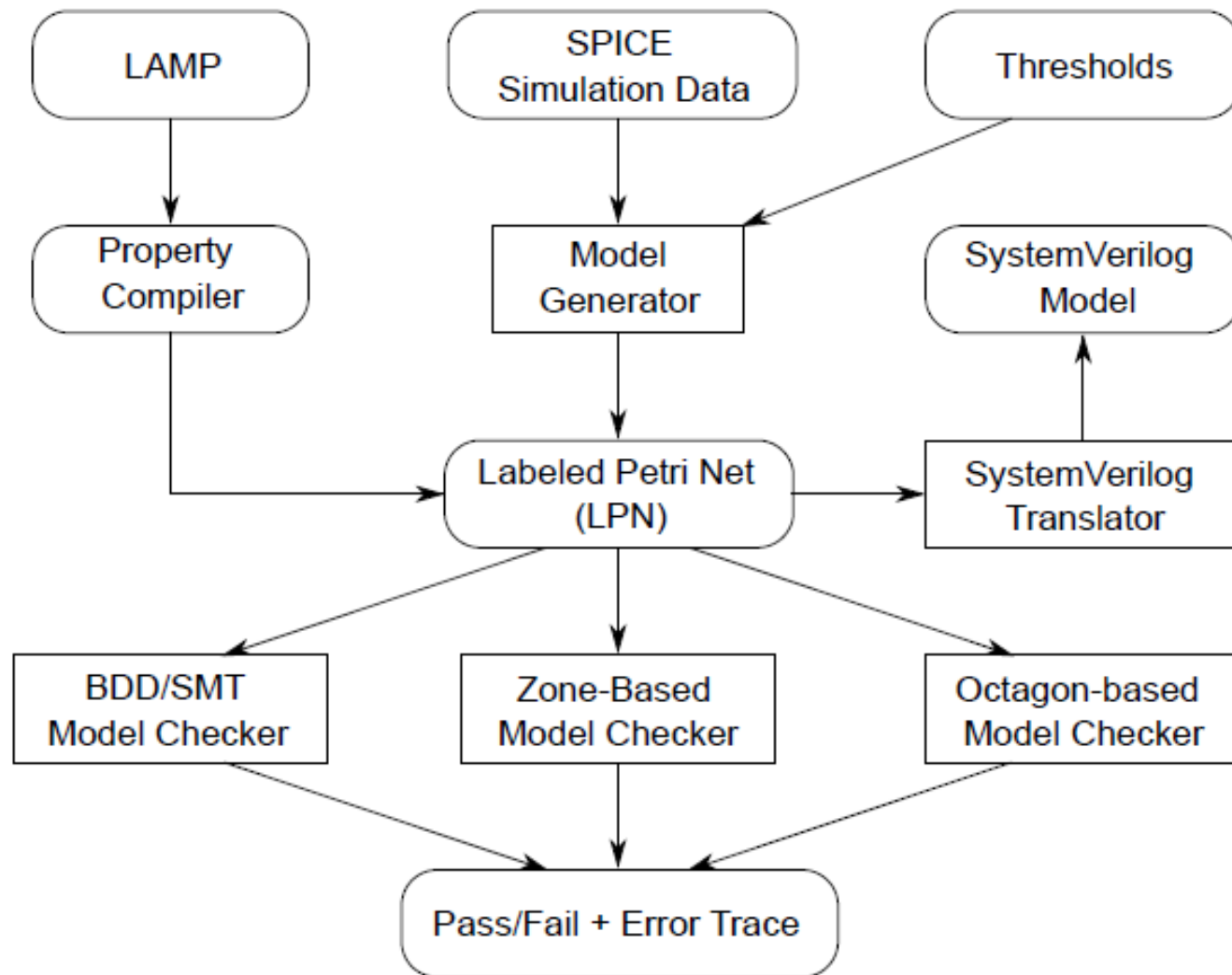
V. Dubikhin, D. Sokolov, A. Yakovlev, and C. J. Myers. Design of mixed-signal systems with asynchronous control. IEEE Design & Test, 33(5):44--55, 2016

LEMA (Myers et al, Utah)

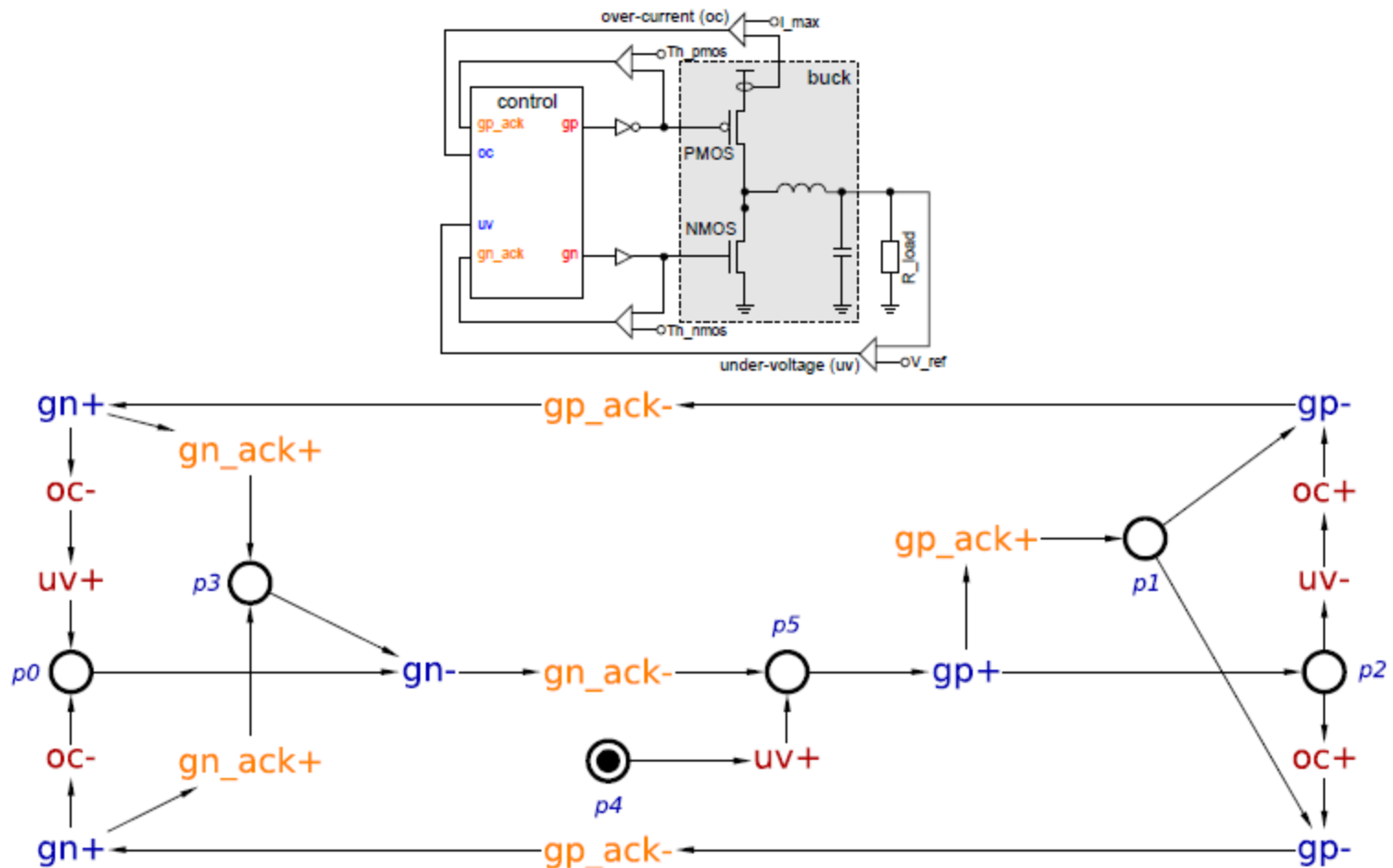
Formal verification of LPN models of AMS circuits, generated from simulation traces.



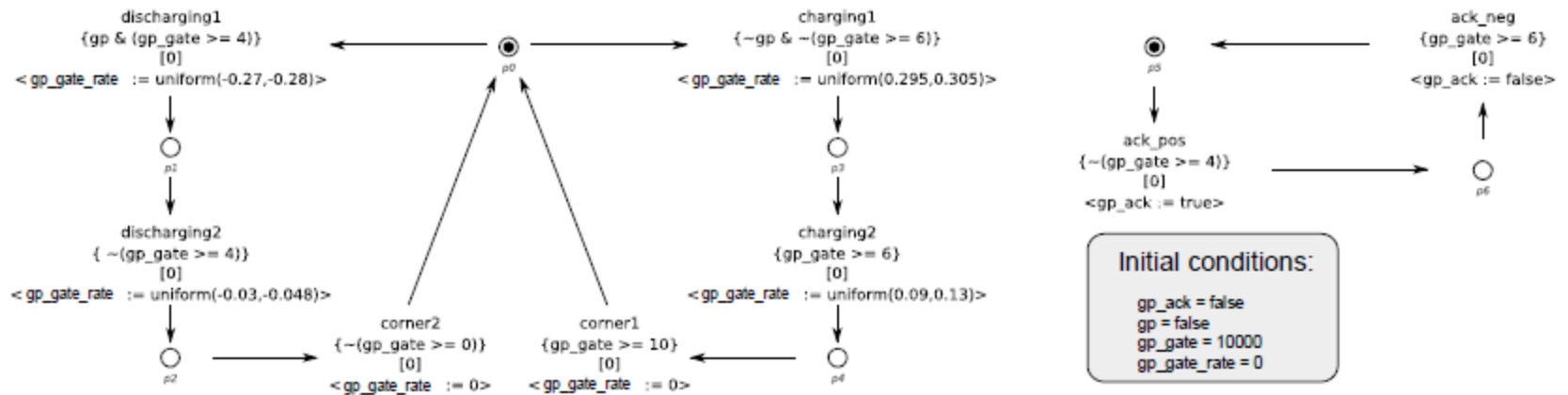
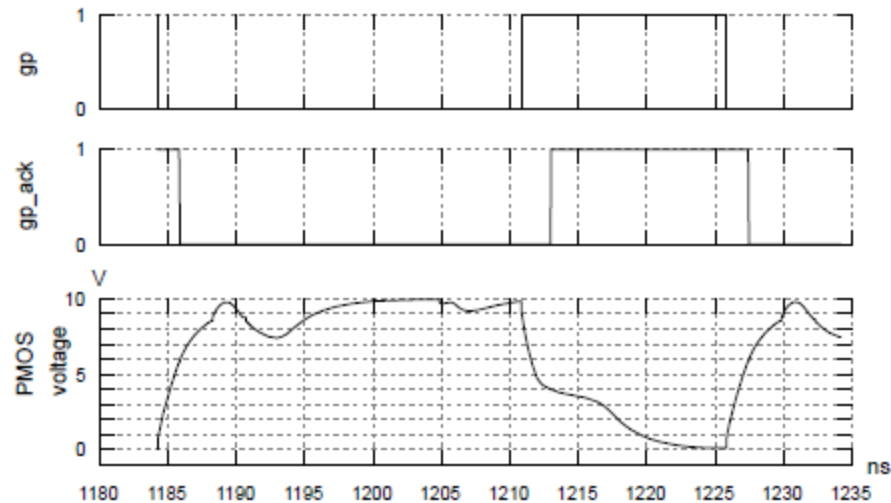
LEMA Tool Flow



Buck converter example

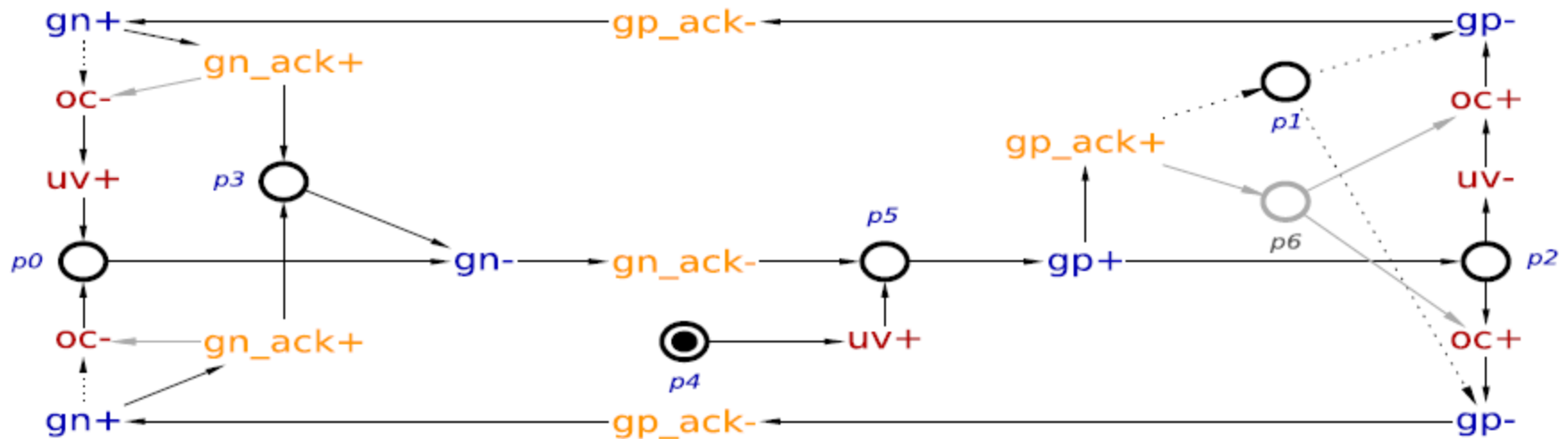


Model generation

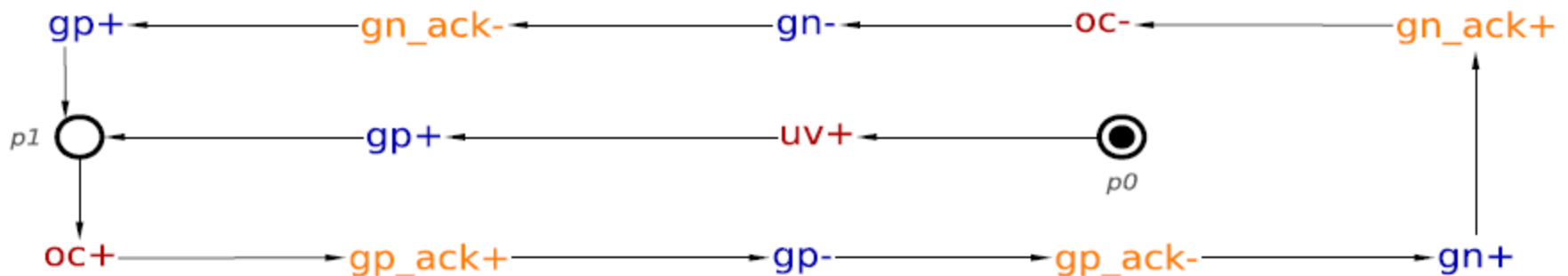


Optimized STG

- Concurrency reduction



- Scenario elimination



Epilogue

Challenges

Messages to take away (for async people)

- **Little digital** circuits can be highly concurrent!
- Asynchronous circuits began their life (in the 50s) for 'little digital' and today is the right time for them
- Analog and mixed-signal is a good application – it combines:
 - Need for **low latency and high range of feedback types**
 - Analog designers are more inclined towards async than digital designers
- **Productivity in industry** is a good drive!
- **Interesting research problems** are there – tech mapping, holistic analog-mixed signal verification, behavioural mining, dealing with complexity
- In particular, **extending the notion of speed-independence** into the world of relative timing, circuits with time comparison (arbitration), with analog components
- Where else do we have little digital now? ... Plastic electronics?

Messages to take away (for async people)

- Little digital circuits can be highly concurrent!
- Asynchronous circuits began their life (in the 50s) for 'little digital' and today is the right time for them
- Analog and digital design are different:
 - Need to think about timing
 - Analog design is more complex
- Design to drive!
- Interesting analog-mixed-signal design is holistic engineering with many constraints
- In particular, the world of asynchronous design is into arbitrage
- Where else can we find asynchronous design? Electronics?

Have a Tryst with an Analog Designer!

Open Challenges

- **Complexity of analog behaviour – model generation/mining**
- **Control stability for AMS with async logic – formal proofs**
- **Electromagnetic effects – break the “circuit theory wall”**
- **How to discretize/quantize best?**
 - **E.g. Make use of the natural spatial (geometric) quantization**
- **How to decompose and distribute computations for AMS systems analysis?**
 - **E.g. use relaxation techniques**

Inspiration comes from

Now, in Maxwell's theory there is the potential energy of the displacement produced in the dielectric parts by the electric force, and there is the kinetic and magnetic energy of the magnetic force in all parts of the field, including the conducting parts. They are supposed to be set up by the current in the wire. **We reverse this**; the current in the wire is set up by the energy transmitted through the medium around it. The energy of the electric machinery ..is transmitting energy from the battery to the wire. **It is definite in amount, and the rate of transmission of energy (total) is also definite in amount.**



Oliver Heaviside (1879)

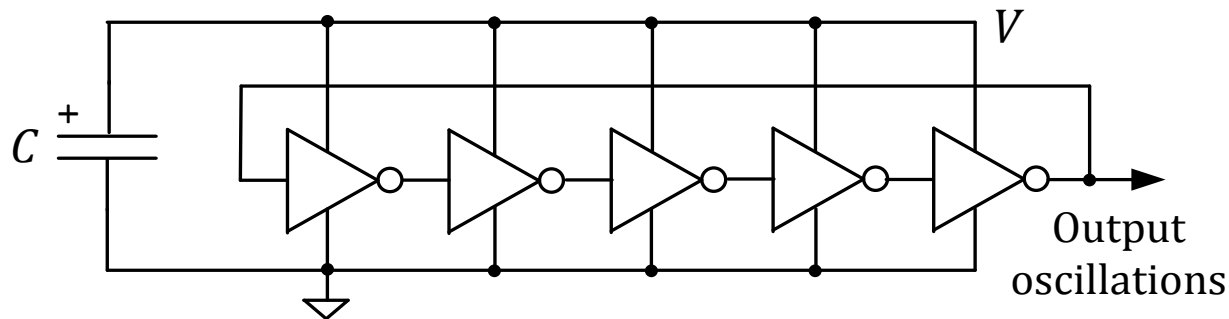
See: O.Heaviside, Electrical papers.
Cambridge University Press, 2011, vol. 2.

Known for many revolutionary
mathematical tools and post-
Maxwell innovations

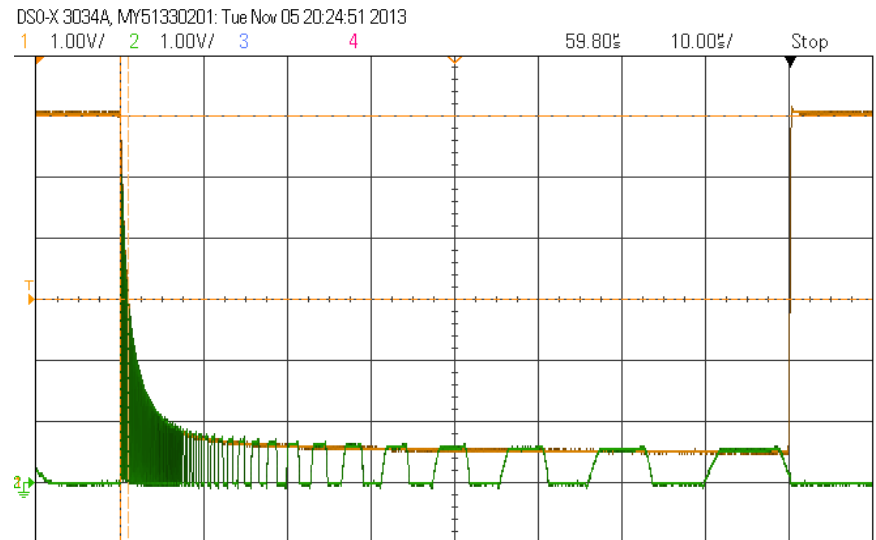
Oliver Heaviside (1850-1925)

Self-taught electrical
engineer, mathematician,
and physicist – who began
as a telegraphy engineer in
Newcastle

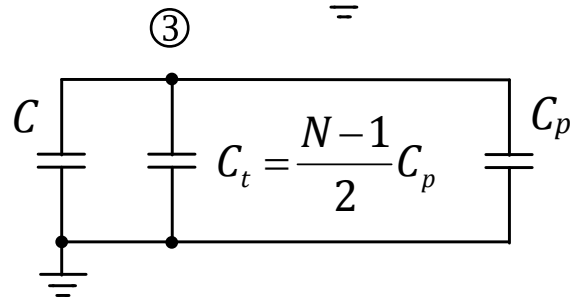
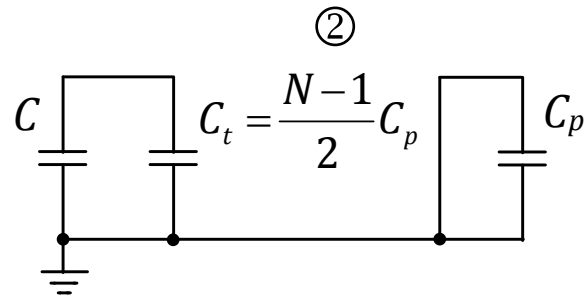
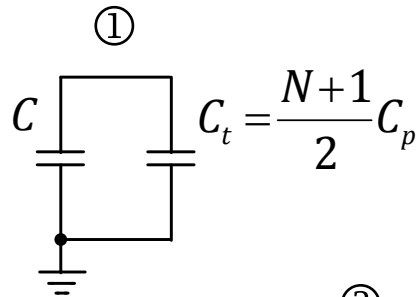
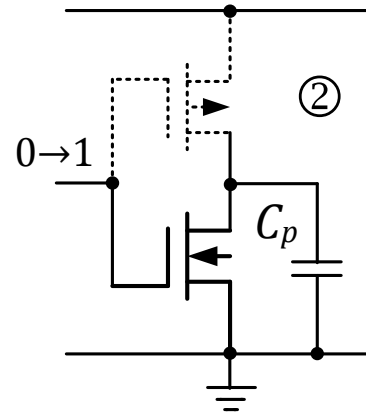
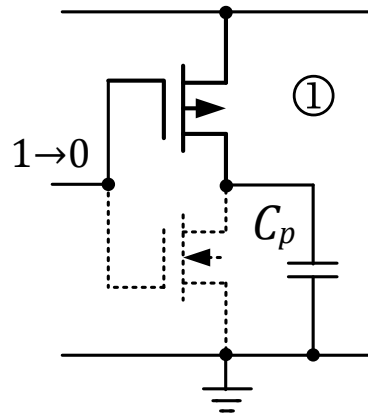
What is Computational Load?



- We employ a simple ring-oscillator to serve as a digital circuit load.
- It is due to the fact that ring-oscillator can closely mimic the switching behaviour of many closed loop self-timed circuits.

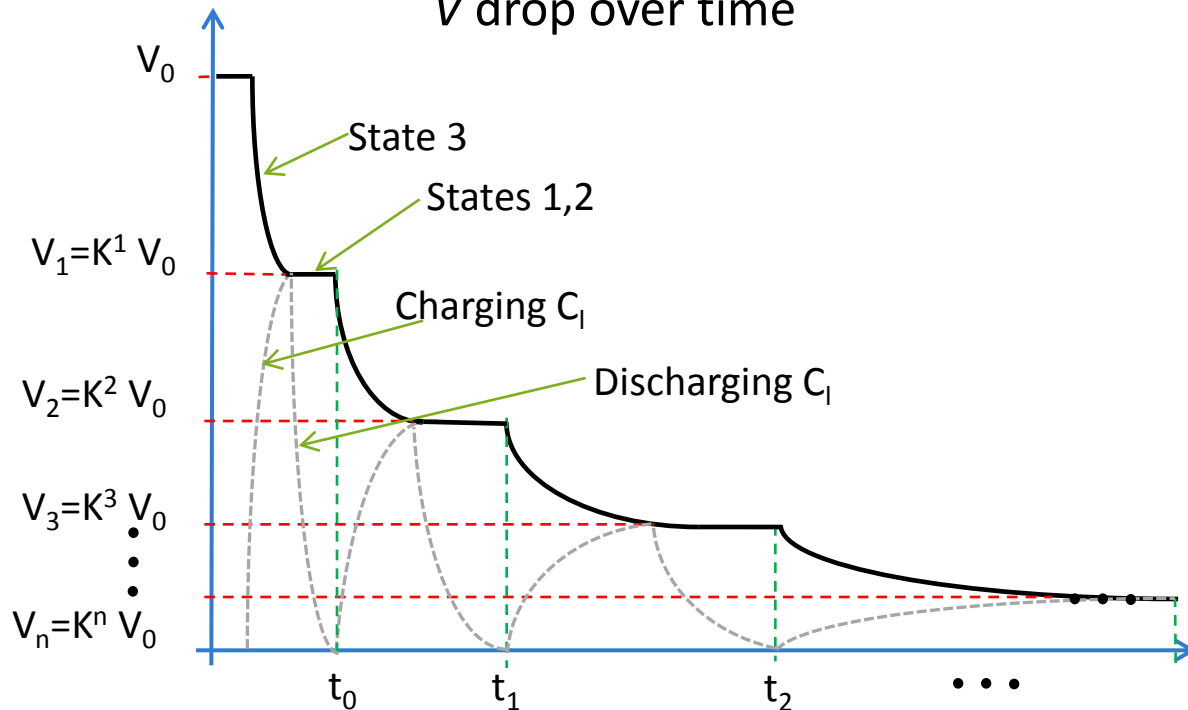


Circuit model



Circuit Model: switching process

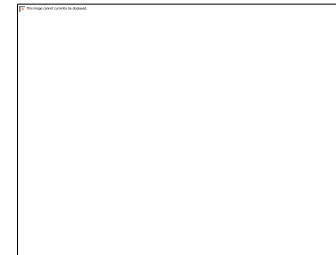
V drop over time



Charge equilibrium at:

$$V_1 = V_0 \frac{C}{C + C_l}$$

$$K = \frac{C}{C + C_l}$$



Solution for Super-threshold

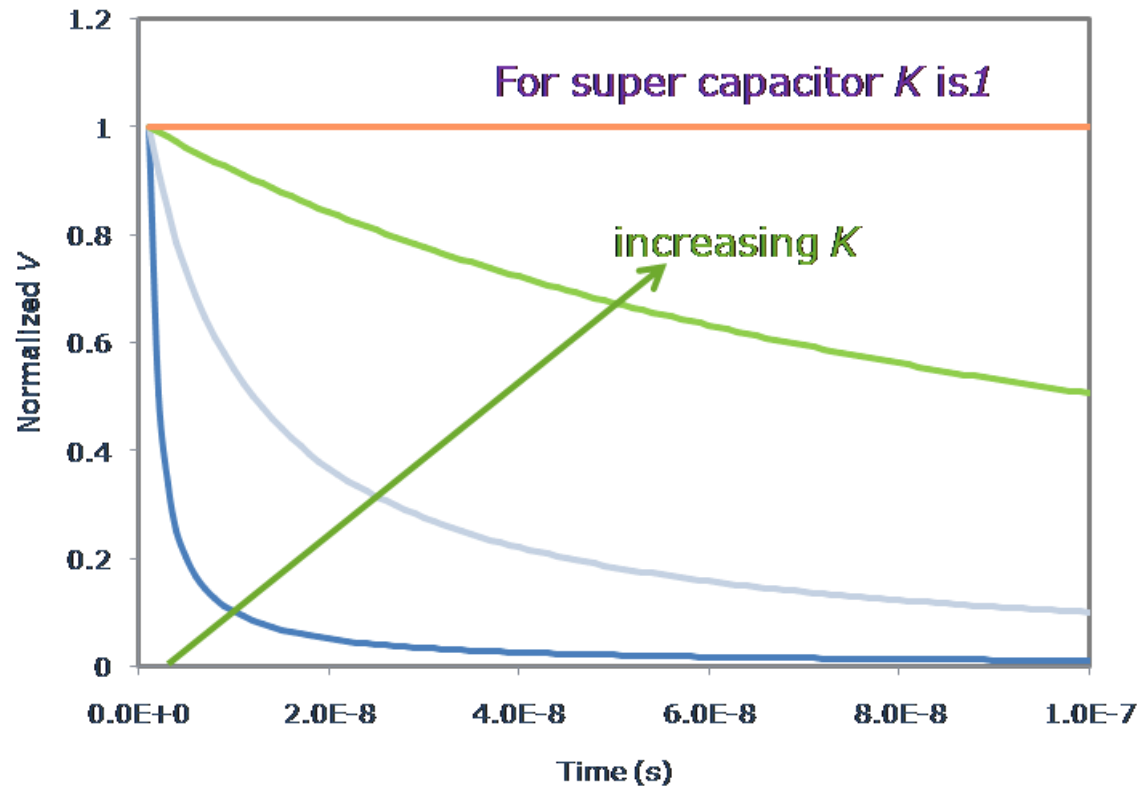
A valid assumption: in super-threshold region we can assume that the propagation delay is inversely proportional to the voltage, so we have:

Switching index	V_N	$t_s = \frac{A}{V}$	Physical time (t)
0	K^0	$\frac{A}{K^0}$	$\frac{A}{K^0}$
1	K^1	$\frac{A}{K^1}$	$\frac{A}{K^0} + \frac{A}{K^1}$
2	K^2	$\frac{A}{K^2}$	$\frac{A}{K^0} + \frac{A}{K^1} + \frac{A}{K^2}$
...			
n	K^n	$\frac{A}{K^n}$	$\sum_{i=0}^n \frac{A}{K^i}$

Solution for Super-threshold

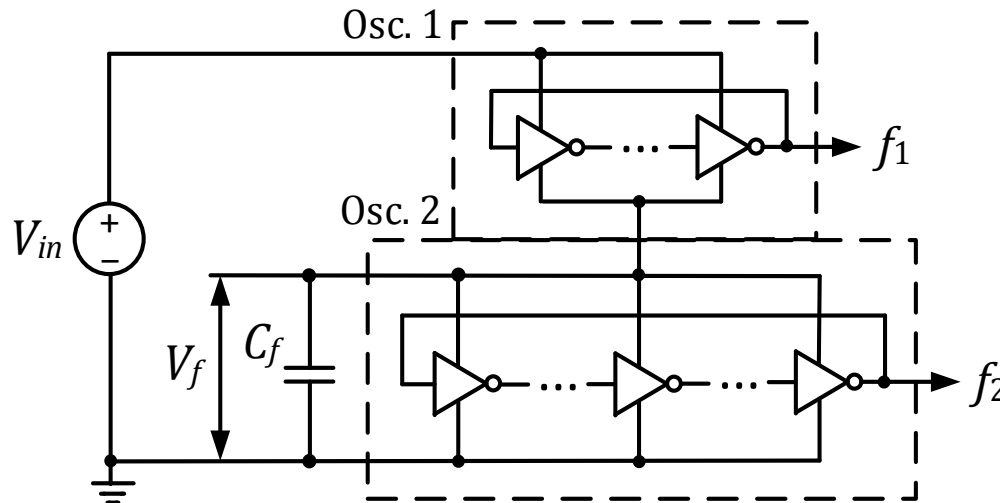
$$V_N = \frac{AK}{t(1-K) + AK}$$

Hyperbolic function of time!



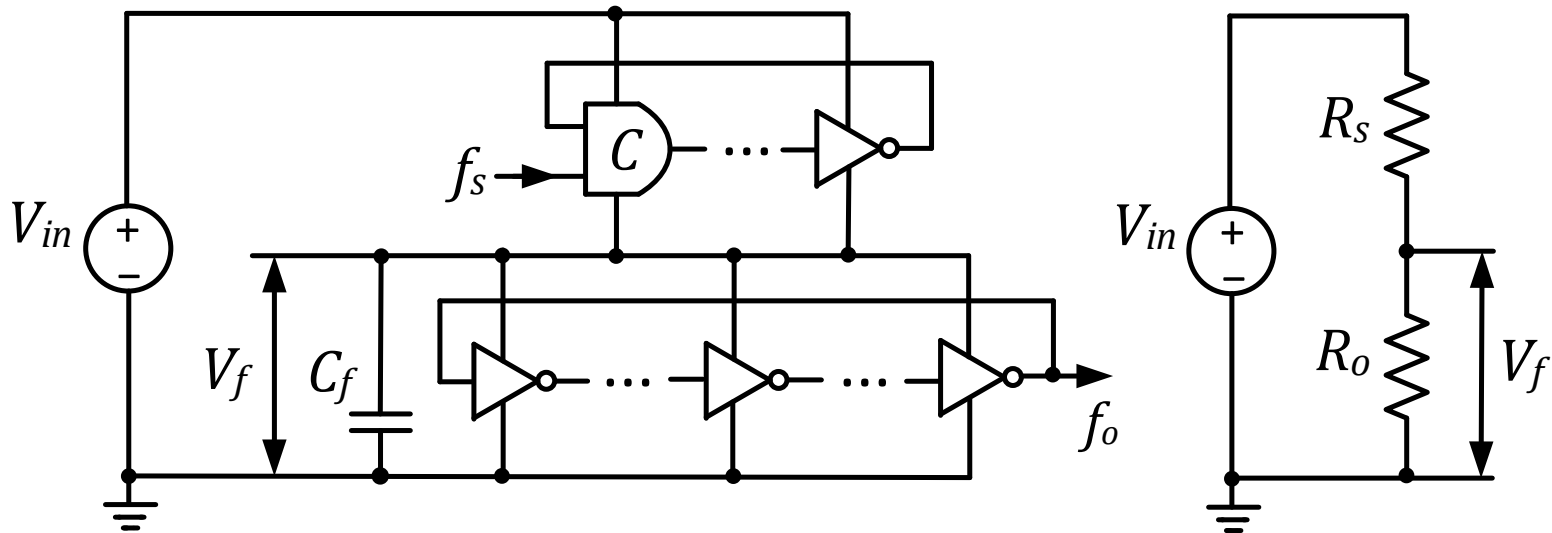
Try stacking digital circuits (ICECS'16)

(Self-)oscillating circuits are voltage-controlled impedances!



- We are talking about DC, i.e. about average value of V_f in steady state
- In practice due to short-circuit currents V_f is close to $0.5V_{in}$ and does not depend on the number of inverters

Frequency mirror (ICECS'16)



Muller C-element: $y = x_1x_2 + (x_1 + x_2)y$

- After some transient the system will reach steady-state defined by

$$R_s = \frac{1}{f_s C_p}; \quad R_o = \frac{1}{f_o C_p}; \quad \frac{V_f}{V_{in} - V_f} = \frac{R_o}{R_s} = \frac{f_s}{f_o}$$

Thanks

- **Newcastle Asynchronous Group:**
 - Victor Khomenko**
 - Danil Sokolov**
 - Andrey Mokhov**
 - Reza Ramezani**
 - Vladimir Dubikhin**
- **Our Collaborators:**
 - Jordi Cortadella (UPC Barcelona)**
 - Chris Myers (University of Utah)**
 - David Lloyd (Dialog Semiconductors)**
 - Alex Kushnerov (BGU, Israel)**
- **EPSRC and Dialog (funding support)**